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Orientiworld

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Abstract: We propose a consistent string theory framework for embedding brane world scenarios with infinite-volume extra dimensions. In this framework the Standard Model fields are localized on D3-branes sitting on top of an orientifold 3-plane. The transverse 6-dimensional space is a non-compact orbifold or a more general conifold. The 4-dimensional gravity on D3-branes is reproduced due to the 4-dimensional Einstein-Hilbert term induced at the quantum level. The orientifold 3-plane plays a crucial role, in particular, without it the D3-brane world-volume theories would be conformal due to the tadpole cancellation. We point out that in some cases the 4-dimensional Planck scale is controlled by the size of certain relevant (as opposed to marginal) orbifold blow-ups. We can then have a scenario with the desirable 4-dimensional Planck scale, the string scale of order TeV, and the cross-over to 10-dimensional gravity around the present Hubble size. We discuss some general features as well as concrete models in this "Orientiworld" framework, including those with D7-branes. We point out that the D7-brane gauge symmetry at the quantum level becomes part of the 4-dimensional gauge symmetry. We present an $\mathcal{N}=1$ supersymmetric model with 3 chiral generations of quarks and leptons, where the original gauge group (which contains an SU(6) subgroup) can be Higgsed to obtain a Pati-Salam model with 3 chiral generations, the Pati-Salam Higgs fields required to break the gauge group further to that of the Standard Model, as well as the desired electroweak Higgs doublets. The superpotential in this model is such that we have precisely one heavy (top-like) generation.

Keywords: Extra Large Dimensions, D-branes, Superstring Vacua.

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1. Introduction

One motivation for considering extra dimensions comes from superstring theory (or M-theory), which is believed to be a consistent theory of quantum gravity. Thus, the critical number of space-time dimensions in superstring theory is 10 (while M-theory is an 11-dimensional theory). However, in order to model the real world with critical string theory (or M-theory), one must address the question of why the extra dimensions have not been observed. One way to make extra dimensions consistent with observation is to assume that they are compact with small enough volume. If the Standard Model gauge and matter fields propagate in such extra dimensions (as is the case in, say, weakly coupled heterotic string theory), then their linear sizes should not be larger than about inverse TeV [1]. On the other hand, in the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space-time. Such a scenario with compact extra dimensions can, for instance, be embedded in superstring theory via type-I$'$ compactifications. Then the extra directions transverse to the branes can be as large as about a tenth of a millimeter [2].

Originally considering compact (or, more generally, finite volume) extra dimensions was motivated by the requirement that at the distance scales for which gravity has been measured one should reproduce 4-dimensional gravity. However, in [3, 4] a remarkable proposal was set forward. In particular, according to [4] 4-dimensional gravity can be reproduced on a 3-brane in infinite-volume bulk (with 6 or larger space-time dimensions) up to ultra-*large* distance scales. Thus, in these scenarios gravity is almost completely localized on a brane (which is almost δ-function-like) with ultra-light modes penetrating into the bulk. As was explained in [4], this dramatic modification of gravity in higher codimension models with infinite volume extra dimensions is due to the Einstein-Hilbert term on the brane, which is induced via loops of non-conformal brane matter [3, 4].

In this paper we describe an explicit string theory framework where the Standard Model gauge and matter fields are localized on (a collection of) D3-branes embedded in infinite-volume extra space. In particular, we consider *unoriented* type-IIB backgrounds in the presence of some number of D3-branes embedded in an orbifolded space-time. The D3-brane world-volume theory in this framework is non-conformal (at least for some backgrounds with at most $\mathcal{N} = 1$ supersymmetry). At the quantum level we have the Einstein-Hilbert term induced on the branes, which leads to almost complete localization of gravity on the D3-branes. In particular, up to an ultra-large cross-over distance scale the gravitational interactions of the Standard Model fields localized on D3-branes are described by 4-dimensional laws of gravity.

One of the key ingredients in this framework is that we have an unoriented background. Thus, if we consider orientable type-IIB backgrounds (that is, those without orientifold planes) in the presence of some number of D3-branes embedded in an orbifolded space-time, then, as was explained in [5], the finiteness of the theory (that is, tadpole cancellation conditions) implies that the D3-brane world-volume theory is necessarily conformal.[1] However, as was originally pointed out in [6], we can obtain *non-conformal* gauge theories if we consider orientifolds of such type-IIB backgrounds. In particular, this is the case for backgrounds with at most $\mathcal{N} = 1$ supersymmetry. Given the importance of orientifold planes in our framework, we refer to it as "Orientiworld".

The orientiworld framework appears to have a rich structure for model building. In particular, since the extra dimensions have infinite volume, the number of D3-branes is arbitrary. Moreover, the number of allowed orbifold groups is infinite. Thus, *a priori* the orbifold group can be an arbitrary[2] subgroup of Spin(6), or, if we require $\mathcal{N} = 1$ supersymmetry to avoid bulk tachyons, of SU(3). To obtain a finite string background, we still must impose *twisted* tadpole cancellation conditions. However, twisted tadpoles must also be canceled in compact type-IIB orientifolds. Then the number of consistent solutions of the latter type is rather limited [7] as we can only have a finite number of D3-branes, and, moreover, the number of allowed orbifold groups is also finite as they must act crystallographically on the compact space. On the other hand, as we already mentioned, in the orientiworld framework the number of consistent solutions is *infinite*, which is encouraging for phenomenologically oriented model building.

This richness of the orientiworld framework can be exploited to construct various models for phenomenological applications. We discuss some examples of orientiworld models in detail. In particular, we give an example with three chiral generations. In this model, which is $\mathcal{N} = 1$ supersymmetric, the gauge group contains a phenomenologically appealing SU(6) subgroup.[3] Albeit the gauge group in this model contains

[1]More precisely, theories with at least $\mathcal{N} = 1$ supersymmetry are conformal [5]. As to non-supersymmetric theories, to decouple bulk tachyons one considers the large-N limit, where they too are conformal.

[2]More precisely, there is a mild restriction on allowed orbifold groups if we require modular invariance of the closed string sector in the corresponding oriented type-IIB background.

[3]A compact version of this model, which is the first orientifold model with three generations of quarks and leptons, was originally constructed in [8] (and a non-compact version was subsequently

an SU(6) subgroup, as was pointed out in [8], it is not a grand unified model as we have no Higgs field in an appropriate higher dimensional representation. However, as was pointed out in [10], one can still break the gauge group down to that of the Standard Model. We discuss this model in detail, and point out the importance of the gauge subgroups coming from D7-branes, which are also present in this model. In particular, the D7-brane gauge symmetry becomes part of the four-dimensional gauge symmetry via the mechanism of [11]. In fact, after a sequence of Higgsing, the gauge group in this model can be broken down to the Pati-Salam gauge group. The charged matter contains 3 chiral generations of quarks and leptons, the electroweak Higgs doublets, as well as the Pati-Salam Higgs fields, which break the Pati-Salam gauge symmetry down to that of the Standard Model. We also point out a pleasant bonus in this model — the superpotential is such that we have precisely one heavy (top-like) generation.

Given recent developments in brane world scenarios with infinite-volume extra dimensions, it is reasonable to expect that the orientiworld approach, which provides an explicit framework for consistently embedding such scenarios in string theory, would have to be further developed. In particular, orbifold examples are only a part of interesting starting type-IIB backgrounds. Thus, one can consider more general backgrounds such as conifolds. It therefore appears that orientiworld might provide a fruitful arena for embedding various brane world scenarios in string theory. In fact, in the following we point out that relevant (as opposed to marginal) blow-ups of the orbifold singularities, which turn the space transverse to the D3-branes into a conifold, play an important role in obtaining 4-dimensional gravity on D3-branes.

The rest of this paper is organized as follows. In section 2 we discuss the orientiworld framework. In section 3 we discuss some general features of the orientiworld scenario, in particular, we discuss how 4-dimensional gravity is reproduced on D3-branes. In section 4 we discuss how the four-dimensional Einstein-Hilbert term arises in the orientiworld context. In section 5 we discuss concrete examples of orientiworld models. In section 6 we discuss some phenomenological aspects of the aforementioned 3-family supersymmetric model containing an SU(6) subgroup. We conclude with final remarks in section 7. Some details are relegated to appendices.

2. Framework

In this section we discuss the orientiworld framework. First we describe the underlying oriented type-IIB orbifold backgrounds. We then consider their orientifolds. This illustrates why introduction of orientifold planes makes a big difference, in particular, why in the orientiworld framework we can construct non-conformal gauge theories. Parts of our discussion here will closely follow [5, 6].

constructed in [9]).

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2.1 Oriented backgrounds

Consider type-IIB string theory with N parallel coincident D3-branes where the space transverse to the D-branes is $\mathcal{M} = \mathbb{R}^6/\Gamma$. The orbifold group $\Gamma = \{g_a \mid a{=}1, \ldots, |\Gamma|\}$ ($g_1 = 1$) must be a finite discrete subgroup of Spin(6) (it can be a subgroup of Spin(6) and not SO(6) as we are dealing with a theory containing fermions). If $\Gamma \subset \mathrm{SU}(3)$ ($\mathrm{SU}(2)$), we have $\mathcal{N} = 1$ ($\mathcal{N} = 2$) unbroken supersymmetry, and $\mathcal{N} = 0$, otherwise.

Let us confine our attention to the cases where type IIB on $\mathcal{M}$ is a modular invariant theory.[4] The action of the orbifold on the coordinates X_i ($i = 1, \ldots, 6$) on $\mathcal{M}$ can be described in terms of SO(6) matrices: $g_a : X_i \to \sum_j (g_a)_{ij} X_j$. (The action of g_a on the world-sheet superpartners of X_i is the same.) We also need to specify the action of the orbifold group on the Chan-Paton charges carried by the D3-branes. It is described by $N \times N$ matrices γ_a that form a representation of Γ. Note that γ_1 is an identity matrix and $\mathrm{Tr}(\gamma_1) = N$.

The D-brane sector of the theory is described by an *oriented* open string theory. In particular, the world-sheet expansion corresponds to summing over oriented Riemann surfaces with arbitrary genus g and arbitrary number of boundaries b, where the boundaries of the world-sheet correspond to the D3-branes. For example, consider the one-loop vacuum amplitude with no handles and two boundaries ($g = 0$, $b = 2$). The corresponding graph is an annulus whose boundaries lie on D3-branes. The one-loop partition function in the light-cone gauge is given by

$$\mathcal{Z} = \frac{1}{2|\Gamma|} \sum_a \mathrm{Tr}\left(g_a (1 - (-1)^F) e^{-2\pi t L_0} \right), \tag{2.1}$$

where F is the fermion number operator, t is the real modular parameter of the annulus, and the trace includes sum over the Chan-Paton factors.

The orbifold group Γ acts on both ends of the open string. The action of $g_a \in \Gamma$ on Chan-Paton charges is given by $\gamma_a \otimes \gamma_a$. Therefore, the individual terms in the sum in (2.1) have the following form:

$$\left(\mathrm{Tr}(\gamma_a)\right)^2 \mathcal{Z}_a \,, \tag{2.2}$$

where $\mathcal{Z}_a$ are characters corresponding to the world-sheet degrees of freedom. The "untwisted" character $\mathcal{Z}_1$ is the same as in the $\mathcal{N} = 4$ theory for which $\Gamma = \{1\}$. The information about the fact that the orbifold theory has reduced supersymmetry is encoded in the "twisted" characters $\mathcal{Z}_a$, $a \neq 1$.

In [5] it was shown that the one-loop massless (and, in non-supersymmetric cases, tachyonic) tadpole cancellation conditions (that is, finiteness of the corresponding string backgrounds) require that

$$\mathrm{Tr}(\gamma_a) = 0 \qquad \forall a \neq 1 \,. \tag{2.3}$$

[4] This is always the case if $\Gamma \subset \mathrm{SU}(3)$. For non-supersymmetric cases this is also true provided that $\nexists \mathbb{Z}_2 \subset \Gamma$. If $\exists \mathbb{Z}_2 \subset \Gamma$, then modular invariance requires that the set of points in $\mathbb{R}^6$ fixed under the $\mathbb{Z}_2$ twist has real dimension 2.

That is, all twisted characters $\mathcal{Z}_a$ contain divergences (which are all massless tadpoles in the supersymmetric cases, while in non-supersymmetric cases we also have tachyonic divergences). In [5] it was also shown that this condition implies that the Chan-Paton matrices γ_a form an n-fold copy of the *regular* representation of Γ. The regular representation decomposes into a direct sum of all irreducible representations $\mathbf{r}_i$ of Γ with degeneracy factors $n_i = |\mathbf{r}_i|$. The gauge group is ($N_i \equiv nn_i$)

$$G = \otimes_i \, \mathrm{U}(N_i) \, . \tag{2.4}$$

The matter consists of Weyl fermions and scalars transforming in bifundamentals $(\mathbf{N}_i, \overline{\mathbf{N}}_j)$ (see [12] for details). The overall center-of-mass U(1) is free — matter fields are not charged under this U(1). All other U(1)'s, however, are running. If $\Gamma \subset \mathrm{SO}(3)$, then these U(1)'s are anomaly free. If, however, $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$, then some of such U(1) factors are actually anomalous (in particular, we have $\mathrm{U}(1)_k \, \mathrm{SU}(N_l)^2$ mixed anomalies), and are broken at the tree-level via a generalized Green-Schwarz mechanism [13, 14]. Apart from the U(1) factors, all supersymmetric theories of this type are conformal [5], in particular, their non-abelian parts are conformal.[5] On the other hand, in non-supersymmetric cases to decouple bulk tachyons one considers the large-N limit.[6] All (supersymmetric as well as non-supersymmetric) gauge theories of this type are conformal in the large-N limit [5]. The key reason for this conformal property is the tadpole cancellation condition (2.3), which, as was explained in [5], implies that all planar diagrams reduce to those of the parent $\mathcal{N} = 4$ theory, which is conformal. In the large-N limit non-planar diagrams are suppressed by powers of $1/N$. For finite-N conformality of the non-abelian parts of the gauge theories follows for $\mathcal{N} = 2$ cases (as $\mathcal{N} = 2$ gauge theories perturbatively are not renormalized beyond one loop), and can also be argued for $\mathcal{N} = 1$ cases [5]. In non-supersymmetric cases, however, bulk tachyons prevent one from considering finite-N cases.[7]

2.2 Unoriented backgrounds

Let us now consider a generalization of the above setup by including orientifold planes. In the following we will mostly be interested in finite-N theories, so let us focus on theories with at least $\mathcal{N} = 1$ unbroken supersymmetry. Thus, consider type-IIB string theory on $\mathcal{M} = \mathbb{C}^3/\Gamma$ where $\Gamma \subset \mathrm{SU}(3)$. Consider the ΩJ orientifold of this theory, where Ω is the world-sheet parity reversal, and J is a $\mathbb{Z}_2$ element ($J^2 = 1$) acting on the complex coordinates z_i ($i = 1, 2, 3$) on $\mathbb{C}^3$ such that the set of points in $\mathbb{C}^3$ fixed under the action of J has real dimension $\Delta = 0$ or 4.

If $\Delta = 0$ then we have an orientifold 3-plane. If Γ has a $\mathbb{Z}_2$ subgroup, then we also have an orientifold 7-plane. If $\Delta = 4$ then we have an orientifold 7-plane. We

[5] We will discuss the U(1) factors in the next subsection.

[6] In this limit we take the string coupling $g_s \to 0$ together with $N \to \infty$ while keeping Ng_s fixed.

[7] In the large-N limit the bulk tachyons are harmless as the string coupling g_s goes to zero.

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may also have an orientifold 3-plane depending on whether Γ has an appropriate $\mathbb{Z}_2$ subgroup. Regardless of whether we have an orientifold 3-plane, we can *a priori* introduce an arbitrary number of D3-branes.[8] On the other hand, if we have an orientifold 7-plane we must introduce 8 of the corresponding D7-branes to cancel the Ramond-Ramond charge appropriately. (The number 8 of D7-branes is required by the corresponding untwisted tadpole cancellation conditions.)

We need to specify the action of Γ on the Chan-Paton factors corresponding to the D3- and D7-branes (if the latter are present, which is the case if we have an orientifold 7-plane). Just as in the previous subsection, these are given by Chan-Paton matrices which we collectively refer to as γ^μ_a, where the superscript μ refers to the corresponding D3- or D7-branes. Note that $\mathrm{Tr}(\gamma^\mu_1) = n^\mu$ where n^μ is the number of D-branes labelled by μ.

At the one-loop level there are three different sources for massless tadpoles: the Klein bottle, annulus and Möbius strip amplitudes depicted in figure 1. The Klein bottle amplitude corresponds to the contribution of unoriented closed strings into the one-loop vacuum diagram. It can be alternatively viewed as a tree-level closed string amplitude where the closed strings propagate between two cross-caps. The latter are the (coherent type IIB) states that describe the familiar orientifold planes. The annulus amplitude corresponds to the contribution of open strings stretched between two D-branes into the one-loop vacuum amplitude. It can also be viewed as a tree-channel closed string amplitude where the closed strings propagate between two D-branes. Finally, the Möbius strip amplitude corresponds to the contribution of unoriented open strings into the one-loop vacuum diagram. It can be viewed as a tree-channel closed string amplitude where the closed strings propagate between a D-brane and an orientifold plane.

Note that there are no Chan-Paton matrices associated with the Klein bottle amplitude since it corresponds to closed strings propagating between two cross-caps which do not carry Chan-Paton charges. The Möbius strip has only one boundary. This implies that the individual terms (corresponding to twists $g_a \in \Gamma$) in the Möbius strip amplitude are proportional to $\mathrm{Tr}(\gamma^\mu_a)$. The annulus amplitude is the same (up to an overall factor of 1/2 due to the orientation reversal projection) as in the oriented case discussed in the previous subsection. Thus, the individual terms (corresponding to twists $g_a \in \Gamma$) in the annulus amplitude are proportional to $\mathrm{Tr}(\gamma^\mu_a)\,\mathrm{Tr}(\gamma^\nu_a)$. The tadpoles can therefore be written as

$$\sum_a \left(K_a + \sum_\mu M^\mu_a \,\mathrm{Tr}(\gamma^\mu_a) + \sum_{\mu,\nu} A^{\mu\nu}_a \,\mathrm{Tr}(\gamma^\mu_a)\,\mathrm{Tr}(\gamma^\nu_a) \right) . \tag{2.5}$$

Here the terms with K_a, M^μ_a and $A^{\mu\nu}_a$ correspond to the contributions of the Klein bottle, Möbius strip and annulus amplitudes, respectively. In fact, the factorization

[8] In general, codimension-3 and higher objects (that is D-branes and orientifold planes) do not introduce untwisted tadpoles.

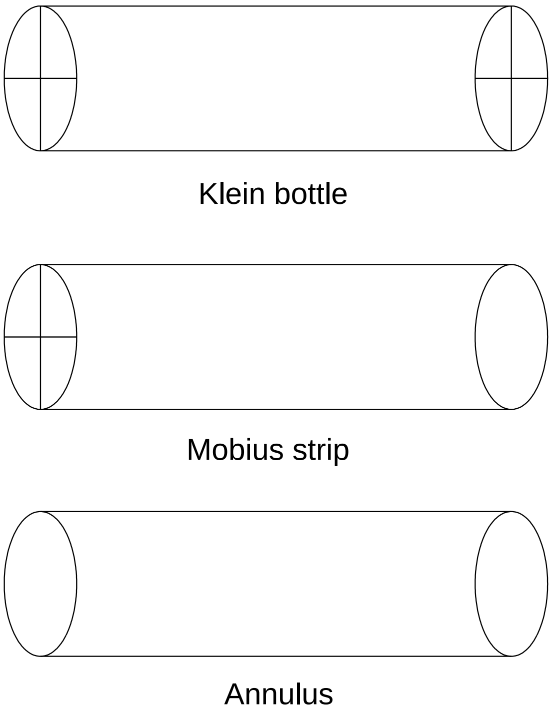

Figure 1: Tree-channel Klein bottle, Möbius strip and annulus amplitudes.

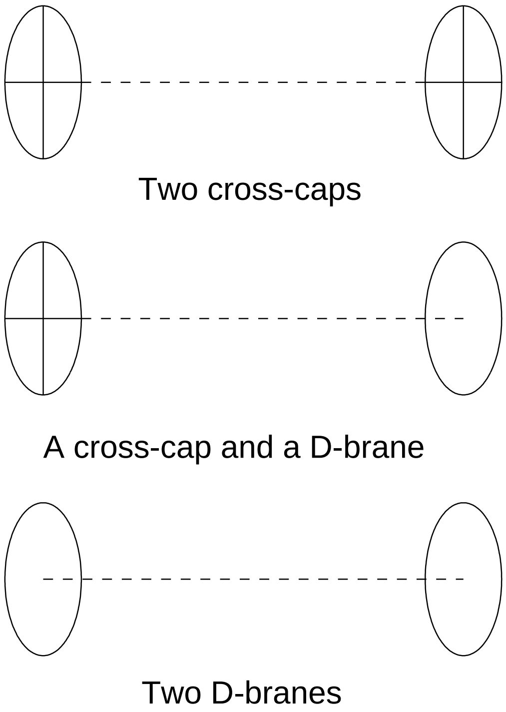

Figure 2: Factorization of the Klein bottle, Möbius strip and annulus amplitudes.

property of string theory implies that the Klein bottle amplitude should factorize into two cross-caps connected via a long thin tube. The Möbius strip amplitude should factorize into a cross-cap and a disc connected via a long thin tube. Similarly, the annulus amplitude should factorize into two discs connected via a long thin tube. These factorizations are depicted in figure 2. The implication of this for the tadpoles is that they too factorize into a sum of perfect squares

$$\sum_a \left(B_a + \sum_\mu C_a^\mu \, \mathrm{Tr}(\gamma_a^\mu) \right)^2 , \tag{2.6}$$

where $B_a^2 = K_a$, $2B_a C_a^\mu = M_a^\mu$ and $C_a^\mu C_a^\nu = A_a^{\mu\nu}$. Thus, the *twisted* tadpole cancellation conditions read:

$$B_a + \sum_\mu C_a^\mu \, \mathrm{Tr}(\gamma_a^\mu) = 0 \, , \qquad a \neq 1 \, . \tag{2.7}$$

These should be contrasted with (2.3) in the oriented case. In particular, since in certain cases some $B_a \neq 0$, *a priori* there is no reason why the corresponding D3-brane world-volume gauge theories should be conformal. That is, the presence of orientifold planes can indeed give rise to non-conformal gauge theories.

2.3 Orientiworld models

Let us see what kind of orientiworld models we can have. In particular, let us discuss what kind of orientiworld models are non-conformal. For definiteness let us focus on the cases where we do have an orientifold 3-plane. If there are no orientifold 7-planes (that is, if Γ does not contain a $\mathbb{Z}_2$ element), then the orientifold projection Ω can be either of the SO or the Sp type: the corresponding orientifold 3-plane is referred to as $\mathrm{O}3^-$ or $\mathrm{O}3^+$, respectively. That is, before orbifolding, if we place $2N$ D3-branes on top of the $\mathrm{O}3^-$-plane ($\mathrm{O}3^+$-plane), we have the $\mathcal{N}=4$ super-Yang-Mills theory with the $\mathrm{SO}(2N)$ ($\mathrm{Sp}(2N)$) gauge group.[9] (We are using the convention where $\mathrm{Sp}(2N)$ has rank N.) After the orbifold projections the 33 (that is, the D3-brane) gauge group is a subgroup of $\mathrm{SO}(2N)$ ($\mathrm{Sp}(2N)$), which can contain $\mathrm{U}(N_k)$ factors as well as SO (Sp) subgroups. The 33 matter can contain bifundamentals in any of these subgroups as well as rank-2 antisymmetric (symmetric) representations in the unitary subgroups. Next, if we have an O7-plane, the orientifold projection Ω must always be of the SO type on the D7-branes — this is required by the tadpole cancellation condition. This, in particular, implies that the 33 and 77 matter cannot contain rank-2 symmetric representations. Note that we also have 37 matter in bifundamentals of the 33 and 77 gauge groups.

Let us start with $\mathcal{N}=2$ theories of this type.[10] All such theories are conformal (more precisely, their non-abelian parts are — here we are ignoring the $\mathrm{U}(1)$ factors). The reason why can be understood as follows. First, note that we can focus on the one-loop level as we have $\mathcal{N}=2$ supersymmetry. It can be shown that cross-cap contributions to the twisted tadpoles corresponding to the twists of even order are all vanishing (see, for instance, [6, 16] and references therein). This implies that the corresponding γ_a^μ are traceless, and, for the same reason as in the previous subsection, do not spoil conformality. Let us therefore focus on orbifold groups $\Gamma \subset \mathrm{SU}(2)$ that do not contain elements of even order. These are given by $\mathbb{Z}_M$ subgroups of $\mathrm{SU}(2)$, where M is odd. Let us see why all such theories are conformal (we can then understand why theories with even order twists are also conformal). In these theories the cross-cap contributions into twisted tadpoles, which are given by the factors B_a, are non-vanishing, so neither are the corresponding $\mathrm{Tr}(\gamma_a)$ (note that in these cases we have no D7-branes, so we do not need to put an additional index μ on the Chan-Paton matrices). However, the tadpole cancellation condition implies that

$$B_a + C_a\,\mathrm{Tr}(\gamma_a) = 0\,, \qquad a \neq 1\,. \tag{2.8}$$

This, as we will see in a moment, implies that these $\mathcal{N}=2$ theories are conformal.

[9] Note that we can also place $2N+1$ D3-branes on top of the $\mathrm{O}3^-$-plane to obtain the $\mathrm{SO}(2N+1)$ gauge group.

[10] Orientifolds of non-compact type-IIB orbifolds were discussed in detail in [6, 15, 16].

To see this, let us consider renormalization of the gauge coupling for a *non-abelian* subgroup of the D3-brane gauge group. This can be deduced from a 2-point function, where two gauge bosons corresponding to this subgroup are attached to the *same* boundary in the corresponding one-loop graph. It has to be the same boundary because these are non-abelian gauge bosons, so the Chan-Paton matrices λ_r, $r = 1, 2$, corresponding to the external lines are traceless: $\mathrm{Tr}(\lambda_r) = 0$. It then follows that $\mathrm{Tr}(\lambda_r \gamma_a) = 0$ as well for by definition λ_r are invariant under the orbifold group action as λ_r correspond to the gauge bosons of the gauge group left unbroken by the orbifold (in particular, note that λ_r commute with γ_a). This then implies that attaching only one such external line to a given boundary would produce a vanishing amplitude. Note that this is correct for the non-abelian gauge bosons, but does not hold for U(1) factors. This, as we will see, is precisely the reason why U(1)'s can still run in these theories.

Thus, the *ultra-violet* divergence structure for the one-loop amplitude containing two external lines corresponding to non-abelian gauge bosons is given by:

$$\sum_a \Big[2B_a C'_a \,\mathrm{Tr}(\lambda_1 \lambda_2 \gamma_a) + 2C_a C'_a \,\mathrm{Tr}(\gamma_a)\,\mathrm{Tr}(\lambda_1 \lambda_2 \gamma_a) \Big] \,, \qquad (2.9)$$

where the first term comes from the Möbius strip, while the second term comes from the annulus. In the second term the factor of 2 is due to the two different ways we can attach external gauge bosons to the two boundaries. The factors B_a and C_a are the same as before — they correspond to the *massless* twisted closed string states coupling to the cross-caps and D3-branes, respectively, in the factorization limit depicted in figure 2. The factors C'_a correspond to these twisted closed string states coupling to the D3-branes with two external lines attached to them (in particular, C'_a are different from C_a). Note that (2.9) vanishes due to the tadpole cancellation conditions (2.8). That is, we have *no* ultra-violet divergences in the one-loop 2-point functions involving non-abelian gauge bosons.

The fact that there are no ultra-violet divergences in the non-abelian 2-point functions does not by itself imply that the non-abelian gauge couplings do not run. Indeed, we must show that there are no *infra-red* divergences in the corresponding 2-point functions either. Remarkably enough, however, the ultra-violet and infra-red divergences in these $\mathcal{N} = 2$ models are related to each other, in fact, they are *identical*. The reason for this is the following. Consider the *loop-channel* one-loop 2-point function involving non-abelian or U(1) gauge bosons (in the latter case we must also include the annulus amplitude with the external lines attached to two different boundaries). Such an amplitude receives contributions only from *massless* open string states running in the loop. This is due to the $\mathcal{N} = 2$ supersymmetry — only BPS states (that is, those in short supermultiplets) can contribute to the renormalization of the gauge couplings, and the massive open string excitations in these backgrounds are all non-BPS [17, 18]. That is, there are no massive string threshold contributions to the gauge couplings, and whatever their field-theoretic logarithmic running, it continues above the string scale. In other words, the massive open string states do *not* provide an ultra-violet cut-off for the gauge coupling running in these backgrounds. This then implies that, if we had logarithmic infra-red divergences in the non-abelian gauge couplings (that is, if the latter did run), we would have to have the corresponding logarithmic ultra-violet divergences as well. However, as we saw above, there are no ultra-violet divergences in the non-abelian gauge couplings. This then implies that there are no infra-red divergences either, and the non-abelian gauge couplings do *not* run — the non-abelian parts of these $\mathcal{N} = 2$ gauge theories are, therefore, conformal.

What about the U(1) gauge couplings? First, note that U(1)'s in these backgrounds are non-anomalous — these theories are $\mathcal{N} = 2$ supersymmetric, and, therefore, non-chiral. So *a priori* they need not be broken. We can then discuss the U(1) gauge coupling running. In fact, all U(1)'s run as there is always matter charged under them.[11] This implies that we have logarithmic *infra-red* divergences in the *loop-channel* one-loop 2-point function involving U(1) gauge bosons. According to the above argument, this then implies that we must also have the corresponding logarithmic *ultra-violet* divergences. And we do. These arise in the annulus amplitude with two external U(1) lines attached to *different* boundaries. In fact, the corresponding infra-red divergence also arises in this amplitude, that is, we have no infra-red divergences in the Möbius strip or annulus amplitudes with two external U(1) lines attached to the same boundary. On the other hand, the aforementioned ultra-violet divergence in the loop channel can be understood as an infra-red divergence in the corresponding tree channel, which arises in the factorization limit depicted in figure 2 due to the exchange of *massless* twisted states (note that massive states do not give rise to infra-red divergences in the tree channel). Such a massless twisted state is a Ramond-Ramond twisted 2-form, call it C, which on top of the four dimensions along the D3-brane world-volumes also propagates in the fixed point locus of the corresponding twist in $\mathbb{C}^3$, which has real dimension 2. This twisted 2-form has a coupling to a U(1) gauge field strength of the following form [19, 20]:

$$\int_{\mathrm{D3}} C \wedge F\,. \tag{2.10}$$

[11] Note that this is different from what happens in oriented backgrounds discussed in subsection 2.1, where we have one center-of-mass U(1) which is free. In the oriented case the gauge group before orbifolding is U(N), and the free center-of-mass U(1) is inherited from this U(N) (while all the other U(1)'s come from breaking the SU(N) part of U(N), so there is always matter charged under these U(1)'s, which, therefore, run). On the other hand, in the unoriented case the gauge group before orbifolding is SO($2N$) or Sp($2N$) (as we have already mentioned, it can also be SO($2N+1$)), and in this case U(1)'s come from breaking these gauge groups, so there is always matter charged under them.

Exchanging this 2-form in the annulus amplitude with two U(1) gauge bosons attached to different boundaries then leads to a logarithmic divergence. The reason why this divergence is logarithmic is simply due to the fact that the massless 2-dimensional euclidean propagator is logarithmic. We will come back to the running of the U(1)'s in a minute. However, let us first understand the loop-channel infra-red divergences (in particular, the absence thereof for non-abelian gauge couplings) in the tree-channel language.

The key point here is that, as we have already mentioned, in these backgrounds the ultra-violet and infra-red divergences in any given one-loop 2-point function are *identical*. Thus, the ultra-violet and infra-red divergences in the annulus amplitude with two non-abelian gauge bosons attached to the same boundary are identical. Similarly, they are identical in the Möbius strip amplitude with two external lines corresponding to non-abelian gauge bosons. Also, the ultra-violet and infra-red divergences are identical in the annulus amplitude with two U(1) gauge bosons attached to the same boundary. Similarly, they are identical in the Möbius strip amplitude with two external lines corresponding to U(1) gauge bosons. Finally, the ultra-violet and infra-red divergences are identical in the annulus amplitude with two U(1) gauge bosons attached to different boundaries. Note that for twists with $\mathrm{Tr}(\gamma_a) \neq 0$ these individual divergences do *not* vanish. In fact, the divergences in the annulus amplitude with two U(1) gauge bosons attached to different boundaries do not vanish even for the twists with $\mathrm{Tr}(\gamma_a) = 0$.

To understand how this is possible, that is, how individual infra-red and ultra-violet divergences in these backgrounds can be identical, let us go to the tree channel. The loop-channel ultra-violet divergences are then translated into the tree-channel infra-red divergences arising due to the exchange of massless twisted states in the factorization limit depicted in figure 2. These divergences are logarithmic as the massless 2-dimensional euclidean propagator is logarithmic. Note that the exchange of massless twisted states in the tree channel also gives rise to logarithmic ultra-violet divergences, which in the loop channel translate into logarithmic infra-red divergences. In fact, precisely because the massless 2-dimensional euclidean propagator is logarithmic, these tree-channel ultra-violet divergences (that is, the loop-channel infra-red divergences) have the *exact same* structure as the tree-channel infra-red divergences (that is, the loop-channel ultra-violet divergences). In particular, this structure is also given by (2.9). What about the exchange of massive twisted states in the tree channel? *A priori* these could also contribute into ultra-violet tree-channel divergences. But they do not. This is also due to the $\mathcal{N} = 2$ supersymmetry. In other words, in $\mathcal{N} = 2$ backgrounds the couplings of massive twisted states to the massless brane fields are such that their exchange in the tree channel does *not* give rise to tree-channel ultra-violet divergences. Indeed, such divergences, if non-vanishing, could only be logarithmic, which would imply that the loop-channel infra-red divergences would be different from the loop-channel ultra-violet divergences. This, however, is not possible as the massive open string states do *not* affect the gauge coupling running in the ultra-violet due to the $\mathcal{N}=2$ supersymmetry as we discussed above.[12]

Let us now go back to the running of the U(1)'s, which is due to the exchange of the massless twisted 2-form fields C. In particular, we have the corresponding logarithmic ultra-violet divergences, which normally are not expected to be present in consistent string backgrounds.[13] Here we would like to discuss a resolution of this point. The idea here is based on the above observation that the ultra-violet and infra-red divergences in these backgrounds go hand-by-hand. In particular, we might expect that, if we remove the infra-red divergences, then this should automatically take care of the ultra-violet divergences as well. This then gives us a hint that, if the U(1)'s become massive, then we should no longer have ultra-violet divergences. In fact, in a moment we will discuss a mechanism for this. The upshot of this mechanism is that these U(1)'s are actually *not* local gauge symmetries but *global* ones, and we should treat them as such. This mechanism is essentially the same in the oriented as well as unoriented backgrounds, so for simplicity we will discuss it in one of the oriented $\mathcal{N}=2$ backgrounds discussed in subsection 2.1.

Thus, consider the oriented background with $\Gamma=\mathbb{Z}_2$, where the generator R of Γ acts as follows on the complex coordinates z_1, z_2, z_3 on $\mathbb{C}^3$:

$$R: z_1 \to z_1\,, \qquad R: z_{2,3} \to -z_{2,3}\,. \tag{2.11}$$

We must satisfy the condition $\mathrm{Tr}(\gamma_R)=0$. Thus, we can take

$$\gamma_R = \mathrm{diag}(I_N, -I_N)\,, \tag{2.12}$$

where I_N is the $N\times N$ unit matrix. The resulting D3-brane world-volume theory is the $\mathcal{N}=2$ supersymmetric $\mathrm{U}(N)\otimes\mathrm{U}(N)$ gauge theory with matter hypermultiplets transforming in

$$(\mathbf{N},\overline{\mathbf{N}})(+1,-1)\,, \qquad (\overline{\mathbf{N}},\mathbf{N})(-1,+1)\,, \tag{2.13}$$

where the U(1) charges are given in the parenthesis. Note that the diagonal U(1), which we will refer to as $\mathrm{U}(1)_+$, is the free center-of-mass U(1), while the anti-diagonal U(1), which we will refer to as $\mathrm{U}(1)_-$, runs.

Consider the Chern-Simons part of the low energy action containing the twisted 2-form C_R [23, 19]:

$$S_{\mathrm{CS}} = \frac{1}{2\pi\alpha'}\int_{\mathrm{D3}} C_R \wedge \mathrm{Tr}\left(\gamma_R\, e^{2\pi\alpha' \mathcal{F}}\right)\,, \tag{2.14}$$

[12]Here we note that these observations are the key ingredient of the Brane-Bulk Duality recently discussed in [21, 22] in the context of non-conformal D3-brane gauge theories from oriented type-IIB backgrounds with uncanceled codimension-2 twisted tadpoles.

[13]In particular, note that we have Landau poles for the U(1)'s.

where $\mathcal{F}$ is the ($(2N)\times(2N)$ matrix valued) D3-brane gauge field strength.[14] Let us decompose $\mathcal{F}$ into ($N\times N$ matrix valued) field strengths $\mathcal{F}_1$ and $\mathcal{F}_2$ for the two $\mathrm{U}(N)$ factors. We then have

$$S_{\mathrm{CS}} = \frac{1}{2\pi\alpha'}\int_{\mathrm{D3}} C_R\wedge\left[\mathrm{Tr}\left(e^{2\pi\alpha'\mathcal{F}_1}\right)-\mathrm{Tr}\left(e^{2\pi\alpha'\mathcal{F}_2}\right)\right]. \tag{2.15}$$

The linear term in the field strengths is given by:

$$\int_{\mathrm{D3}} C_R\wedge F_-\,, \tag{2.16}$$

where

$$F_-\equiv \mathrm{Tr}(\mathcal{F}_1)-\mathrm{Tr}(\mathcal{F}_2) \tag{2.17}$$

is the (appropriately normalized) $\mathrm{U}(1)_-$ field strength. In particular, note that C_R does not have such a coupling to the $\mathrm{U}(1)_+$ field strength F_+. This is consistent with the fact that the latter does not run.

We can consistently give a mass to the $\mathrm{U}(1)_-$ gauge field as follows. Let $\overline{C}_R$ be the pull-back of the 2-form C_R onto the D3-brane world-volume:

$$\overline{C}_R^{\mu\nu} = \delta^\mu{}_M\delta^\nu{}_N C_R^{MN}\big|_{\mathrm{D3}}\,. \tag{2.18}$$

Here x^μ are the coordinates along the D3-brane world-volume, while x^i, $i=1,2$, are the real coordinates on the fixed point locus of the twist R ($z_1=x^1+ix^2$), and $x^M=(x^\mu,x^i)$. Note that the Chern-Simons coupling (2.16) is given by

$$\int_{\mathrm{D3}} d^4x\;\epsilon_{\mu\nu\sigma\rho}\overline{C}_R^{\mu\nu}F_-^{\sigma\rho}\,. \tag{2.19}$$

Next, let $\overline{H}_R$ be the *four-dimensional* field strength of $\overline{C}_R$. That is,

$$\overline{H}_R^{\mu\nu\sigma} = \partial^\mu\overline{C}_R^{\nu\sigma}+\partial^\nu\overline{C}_R^{\sigma\mu}+\partial^\sigma\overline{C}_R^{\mu\nu}\,. \tag{2.20}$$

Note that we can add a kinetic term for $\overline{C}_R$ in the D3-brane world-volume, which is consistent with all symmetries of the system:

$$-\frac{L^2}{12}\int_{\mathrm{D3}}\overline{H}_R^2\,. \tag{2.21}$$

Here L has the dimension of length. The original action is restored in the $L\to 0$ limit. In this limit, however, the $\mathrm{U}(1)_-$ gauge boson is infinitely heavy. Indeed, consider the action (g_- is the $\mathrm{U}(1)_-$ gauge coupling):

$$\begin{aligned}-\frac{L^2}{12}\int_{\mathrm{D3}}\overline{H}_R^2+\int_{\mathrm{D3}}\overline{C}_R\wedge F_- -\frac{1}{4g_-^2}\int_{\mathrm{D3}}F_-^2 &=\\ =-\frac{L^2}{12}\int_{\mathrm{D3}}\overline{H}_R^2-\frac{2}{3}\int_{\mathrm{D3}}\overline{H}_R\wedge A_- -\frac{1}{4g_-^2}\int_{\mathrm{D3}}F_-^2\,,&\end{aligned} \tag{2.22}$$

[14] Note that we are ignoring the NS-NS 2-form, which is not important for our discussion here. At any rate, it does not appear in the unoriented backgrounds.

where A_- is the $\mathrm{U}(1)_-$ gauge field. The mass of the $\mathrm{U}(1)_-$ gauge boson therefore is

$$m_-^2 = \frac{16 g_-^2}{L^2}\,. \tag{2.23}$$

Thus, for non-zero L the $\mathrm{U}(1)_-$ gauge symmetry is broken. However, we still have a *global* $\mathrm{U}(1)_-$ symmetry.

Since $\mathrm{U}(1)_-$ is heavy for non-vanishing L, we have no infra-red divergences. Let us, however, check that our expectation about the absence of the ultra-violet divergences is actually correct. What we need to do is to compute the correction to the F_-^2 kinetic term due to the tree-channel exchange of the 2-form C_R. The relevant action reads:

$$-\frac{L^2}{12}\int_{\mathrm{D3}} \overline{H}_R^2 + \int_{\mathrm{D3}} \overline{C}_R \wedge F_- - \frac{1}{4g_-^2}\int_{\mathrm{D3}} F_-^2 - \frac{1}{12}\int_{\mathrm{D3}\times\mathbb{R}^2} H_R^2\,, \tag{2.24}$$

where H_R is the field strength of the 2-form C_R in the bulk, which is the D3-brane world-volume times the fixed point locus of the twist R. The computation of this tree-channel exchange is given in appendix A. As can be seen from this computation, we indeed do not have logarithmic divergences in the $\mathrm{U}(1)_-$ gauge coupling even if we take the limit $L \to \infty$ where $m_- \to 0$. The physical interpretation of this is that $\mathrm{U}(1)_-$ should *not* be interpreted as a local gauge symmetry but as a *global* one. In particular, suppose we start from the original background with a massless $\mathrm{U}(1)_-$ gauge boson. Since this $\mathrm{U}(1)_-$ runs, we must introduce an ultra-violet cut-off Λ. Since the 2-form C_R couples to the brane fields via (2.15), already at the one-loop level a kinetic term (2.21) for C_R will be generated on the D3-branes. At the one-loop order the relevant couplings in (2.15) are those quadratic in the field strengths:

$$\pi\alpha' \int_{\mathrm{D3}} C_R \wedge \left[\mathrm{Tr}(\mathcal{F}_1\mathcal{F}_1) - \mathrm{Tr}(\mathcal{F}_2\mathcal{F}_2)\right]. \tag{2.25}$$

In fact, the coefficient L^2 in (2.21) is given by:

$$L^2 = b(\alpha')^2\Lambda^2\,, \tag{2.26}$$

where b is a dimensionless numerical coefficient. The computation of appendix A then shows that we do not have a logarithmic divergence in the $\mathrm{U}(1)_-$ gauge coupling even in the $\Lambda \to \infty$ limit. This, as we have already mentioned, suggests that $\mathrm{U}(1)_-$ should be treated as a global symmetry and not as a local one. In fact, in appendix A we point out that this is consistent with the case where the two extra dimensions x^i are compactified on a large 2-torus.

As we have already mentioned, the story in unoriented backgrounds is similar. Once we interpret all U(1)'s as global symmetries, we are left with $\mathcal{N} = 2$ superconformal non-abelian gauge theories on D3-branes. As should be clear from our discussions, the key reason why $\mathcal{N} = 2$ theories are conformal lies in the fact that

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all non-trivial twists have fixed point loci of real dimension 2. In particular, the fact that the ultra-violet and infra-red divergences in these backgrounds are identical is possible precisely because they are identical in a massless 2-dimensional euclidean propagator.[15] This gives us a hint that all $\mathcal{N}=1$ theories with the orbifold groups $\Gamma \subset \mathrm{SO}(3)$ should also be conformal. Indeed, each individual twist $g_a \in \Gamma$ in these cases has a fixed point locus of real dimension 2, moreover, each individual twist $g_a \in \mathrm{SU}(2)$, so it preserves $\mathcal{N}=2$ supersymmetry (however, at least two different twists belong to two *different* SU(2)'s, so altogether we have only $\mathcal{N}=1$ supersymmetry as $\Gamma \subset \mathrm{SO}(3) \subset \mathrm{SU}(3)$). It is then not difficult to see that the one-loop non-abelian β-functions vanish in these theories. Clearly, since these theories are not $\mathcal{N}=2$ supersymmetric, the one-loop vanishing of the corresponding β-functions does not guarantee that they are conformal at higher loops. Actually, it is not difficult to show that in such theories even two-loop non-abelian β-functions vanish. Indeed, this would follow from the vanishing of the one-loop β-functions plus the vanishing of the one-loop anomalous scaling dimensions for the non-abelian matter fields. The latter fact can be shown by repeating the above arguments for matter fields. In fact, in the $\mathcal{N}=1$ cases one can argue that these theories are conformal (even at finite N) by adapting the corresponding arguments given in [5] in the case of oriented theories.

Before we move onto other $\mathcal{N}=1$ theories, let us note the following about $\mathcal{N}=1$ theories with $\Gamma \in \mathrm{SO}(3)$. First, the U(1) factors are all anomaly free in such theories (and should be treated as global symmetries just as in the $\mathcal{N}=2$ cases). This, actually, follows from the fact that all such theories are *non-chiral*. Since such theories are not phenomenologically appealing, regardless of their conformality we would have to move onto $\mathcal{N}=1$ theories with $\Gamma \not\subset \mathrm{SO}(3)$. As we will now discuss, such theories are not conformal.

2.4 Non-conformal theories

Thus, consider the cases where $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$. It then follows that $\exists g_a \in \Gamma$ such that the corresponding fixed point loci have real dimension 0. Let us focus on such twists, and discuss their contributions to the one-loop running of the gauge couplings. Let us first discuss the loop-channel ultra-violet divergences, which translate into the tree-channel infra-red divergences due to the exchange of massless twisted states corresponding to such twists. These massless divergences are no longer logarithmic but *quadratic* (in energy or inverse length — in terms of the real modular parameter on the annulus the corresponding divergence is of the form $\int dt/t^2$ at $t \to 0$). This can be understood from the fact that the twisted closed string states propagating in the tree channel live at the fixed *point*. Then, if the external lines correspond to *massless* on-shell states, the massless twisted state in

[15] As we discussed above, this *plus* $\mathcal{N}=2$ supersymmetry are the key ingredients here — the latter ensures that the massive states do not contribute.

the tree-channel exchange is also on-shell, and its propagator $1/p^2$ is quadratically divergent. Note, however, that these quadratic divergences cancel not only in the vacuum amplitude but also in the two-point one-loop amplitudes with two non-abelian gauge bosons attached to the same boundary. As in the previous subsection, this cancellation occurs between the annulus and Möbius strip amplitudes. In fact, the quadratic divergence structure is precisely of the form (2.9). This then implies that all such quadratic divergences cancel. This is just as well — in consistent backgrounds we should have no quadratic divergences in a four-dimensional non-abelian gauge boson propagator. In fact, such quadratic divergences also cancel in all other 2-point amplitudes involving non-abelian matter fields as they should in a consistent $\mathcal{N}=1$ supersymmetric theory.

As should be clear from our previous remarks, the same, however, does not hold for U(1) factors. In fact, for the models with $\Gamma\subset \mathrm{SU}(3)$ but $\Gamma\not\subset \mathrm{SO}(3)$ we always have at least one U(1) factor in the D3-brane world-volume. Let us therefore consider 2-point functions involving such a U(1) factor. Thus, at the one-loop level, for the diagrams where we attach both external lines corresponding to a U(1) gauge boson to the same boundary, we have a cancellation between the annulus and Möbius strip amplitudes just as in the case of non-abelian gauge bosons. There is, however, another diagram we must consider, namely, the annulus diagram with the two external lines attached to *different* boundaries. (Note that we do not have the corresponding Möbius strip amplitude as the latter has only one boundary.) This annulus amplitude *does* contain a quadratic divergence due to a massless twisted closed string exchange. Thus, we have at least one Ramond-Ramond twisted 2-form, call it C, living at the fixed *point* in $\mathbb{C}^3$ (recall that in these models we always have a twist with a fixed point locus of real dimension 0). This twisted 2-form has a coupling to the U(1) field strength of the following form:

$$\int_{\mathrm{D3}} C\wedge F\,. \tag{2.27}$$

Exchanging this 2-form in the annulus amplitude with two U(1) gauge bosons attached to different boundaries leads to a quadratic divergence.

The presence of this quadratic divergence is due to the fact that at least one U(1) factor in such backgrounds is actually *anomalous* [26, 27, 13]. In particular, we have a cubic $\mathrm{U}(1)^3$ anomaly (note that all such theories are actually *chiral*). The anomalous U(1)'s in such backgrounds are broken at the tree level via a generalized Green-Schwarz mechanism involving twisted closed string states [13]. In particular, we now have a tree-level kinetic term for the twisted 2-forms C, which propagate in the D3-brane world-volume. The anomalous U(1)'s then become massive, and we have the corresponding global U(1) symmetries at the orbifold point (that is, if we do not blow up the orbifold[16]). This then takes care of the aforementioned quadratic

[16]As was pointed out in [27, 28], there appear to be some subtleties in the world-sheet description

divergences. In fact, to begin with these divergences appeared because of an artificial separation of the physical degrees of freedom in massive U(1)'s.

Let us now turn to the loop-channel infra-red divergences, which translate into the tree-channel ultra-violet divergences. In particular, let us discuss such divergences in the non-abelian gauge couplings. The key point here is that, unlike in the $\mathcal{N} = 2$ models discussed in the previous subsection, here the massive open string states do contribute into the gauge coupling renormalization (in particular, we have massive string threshold corrections). In fact, they provide an effective ultra-violet cut-off in the loop channel [24, 25]. There are, however, *infra-red* divergences in the loop channel. Once again, unlike in the $\mathcal{N} = 2$ models, here the ultra-violet and infra-red divergence structures are *different*. In particular, in the tree channel the massive twisted states now do contribute into ultra-violet divergences. In the loop channel this translates into *logarithmic* infra-red divergences corresponding to the non-abelian gauge coupling running. That is, all models with $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$ are actually *non-conformal*. In particular, the one-loop β-function coefficients in these theories are non-vanishing [6, 9]. Thus, we can obtain D3-brane theories which are both chiral and non-conformal if we consider orientiworld models with $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$.

Before we end this subsection, let us make the following observations. Let us assume that we have an O3-plane. In the cases where Γ contains a $\mathbb{Z}_2$ element we also have the corresponding O7-plane as well as D7-branes. At the *tree level* the D7-brane gauge symmetry serves as a *global* symmetry for the D3-brane gauge theory. In particular, we have 37 bifundamental matter, which, from the four-dimensional viewpoint, appears as matter transforming in the fundamental representations of various 33 gauge subgroups that also carries global quantum numbers with the corresponding global symmetry identified with the 77 gauge symmetry. However, already at the one-loop level we have kinetic terms for the 77 gauge bosons generated in the D3-brane world-volume. That is, the full four-dimensional gauge group contains a subgroup corresponding to the original 77 gauge group. More precisely, in chiral models with $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$ we have anomalous 77 U(1)'s on the D3-branes. These U(1)'s are then broken. However, the non-abelian part of the 77 gauge group becomes part of the four-dimensional gauge symmetry.

3. Gravity in orientiworld

One interesting feature of the orientiworld framework we described in the previous section is that we can construct models with non-conformal gauge theories in the D3-brane world-volumes. Following [3, 4] we then expect that the Einstein-Hilbert

in such backgrounds at the orbifold point in the moduli space. However, as was discussed in [27, 28], the world-sheet description is adequate if we blow up the orbifold fixed point. We will come back to this point in section 4.

term (among other terms) will be generated on the D3-branes[17] (we will discuss this point in more detail in the next section). Then, since D3-branes are codimension-6 objects, following [4], we expect almost completely localized gravity on D3-branes with ultra-light modes penetrating into the bulk. In this section we would like to discuss this phenomenon in the orientiworld context.

To understand how such dramatic modification of gravity occurs in the orientiworld context, we will first consider a simplified model. In this model the 3-brane on which the Standard Model fields are localized is assumed to be tensionless. This assumption, however, is not expected to affect the main conclusions of our discussion. In fact, recently in [32] it was argued in the case of non-zero tension codimension-2 branes that almost complete localization of gravity still takes place on such branes. We then may expect the same to hold for higher codimension branes as well.[18]

3.1 A toy model

In this subsection we discuss a toy brane world model, which captures the key features of gravity in the orientiworld framework, with a codimension-d brane ($d \geq 2$) embedded in a D-dimensional bulk space-time. (For calculational convenience we will keep the number of space-time dimensions D unspecified, but we are mostly interested in the case $D = 10$ and $d = 6$, where the brane is a 3-brane.) The action for this model is given by:

$$S = \widehat{M}_P^{D-d-2} \int_\Sigma d^{D-d}x \sqrt{-\widehat{G}}\, \widehat{R} + M_P^{D-2} \int d^D x \sqrt{-G}\, R\,. \tag{3.1}$$

Here M_P is the (reduced) $D-$dimensional Planck mass, while $\widehat{M}_P$ is the (reduced) $(D-d)$-dimensional Planck mass; Σ is a δ-function-like codimension-d source brane, which is a hypersurface $x^i = x^i_*$ (x^i, $i = 1, \dots, d$, are the d spatial coordinates transverse to the brane); the brane is assumed to be tensionless; also,

$$\widehat{G}_{\mu\nu} \equiv \delta_\mu{}^M \delta_\nu{}^N G_{MN}\big|_\Sigma\ , \tag{3.2}$$

where x^μ are the $(D-d)$ coordinates along the brane (the D-dimensional coordinates are given by $x^M = (x^\mu, x^i)$, and the signature of the D-dimensional metric is $(-,+,\dots,+)$); finally, the $(D-d)$-dimensional Ricci scalar $\widehat{R}$ is constructed from the $(D-d)$-dimensional metric $\widehat{G}_{\mu\nu}$, and the Einstein-Hilbert term on the brane is assumed to be generated via quantum loops of matter localized on the brane [3, 4],

[17] The fact that the Einstein-Hilbert term can be present on D-branes was originally suggested in [29]. A string theory computation of such a term was recently given in [30] in a somewhat different context. A discussion of the Einstein-Hilbert term on D-branes can also be found in [31].

[18] In the higher codimension case, if we take a non-zero tension 3-brane to be δ-function-like, the background is badly singular at the location of the brane. However, as we will see in the following, it appears that the brane thickness should be assumed to be non-zero (albeit it can be small), which smoothes out such singularities — see below.

which is assumed to be non-conformal. Note that this is nothing but the Dvali-Gabadadze model [4], except that we will not assume that the extra dimensions transverse to the brane are flat.

The equations of motion following from the action (3.1) are given by:

$$R_{MN} - \frac{1}{2}G_{MN}R + \frac{\sqrt{-\widehat{G}}}{\sqrt{-G}}\delta_M{}^\mu\delta_N{}^\nu\left[\widehat{R}_{\mu\nu} - \frac{1}{2}\widehat{G}_{\mu\nu}\widehat{R}\right]L^d\delta^{(d)}(x^i - x^i_*) = 0\,, \qquad (3.3)$$

where $\widehat{R}_{\mu\nu}$ is the $(D-d)$-dimensional Ricci tensor constructed from the metric $\widehat{G}_{\mu\nu}$, and $L^d \equiv \widehat{M}_P^{D-d-2}/M_P^{D-2}$.

Consider the following ansätz for the metric

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + \widetilde{G}_{ij}dx^i dx^j\,, \qquad (3.4)$$

where $\widetilde{G}_{ij}$ is a function of x^i but is independent of x^μ. Let $\widetilde{R}_{ij}$ and $\widetilde{R}$ be the d-dimensional Ricci tensor respectively Ricci scalar constructed from the d-dimensional metric $\widetilde{G}_{ij}$. With the above ansätz we have that the d-dimensional manifold $\mathcal{M}_d$ transverse to the brane must be Ricci flat:

$$\widetilde{R}_{ij} = 0\,. \qquad (3.5)$$

In particular, it need not be flat ($\mathcal{M}_d = \mathbb{R}^d$ is a special case of a Ricci-flat space, in particular, for high enough d there exist Ricci-flat manifolds other than $\mathbb{R}^d$). We will, however, assume that $\mathcal{M}_d$ is non-compact (and has infinite volume).

Ricci flat manifolds are precisely the spaces that arise in the orientiworld construction (in the weak string coupling regime). Examples of such spaces are given by conifolds. Thus, let[19] $d > 2$, and let the metric on $\mathcal{M}_d$ be given by

$$d\widetilde{s}^2 = dr^2 + r^2\gamma_{\alpha\beta}dy^\alpha dy^\beta\,, \qquad (3.6)$$

where $\gamma_{\alpha\beta}$ is independent of the "radial" coordinate r (r takes values between 0 and ∞), and is a metric on a (compact) $(d-1)$-dimensional manifold $\mathcal{Y}_{d-1}$ of positive curvature. Then the manifold $\mathcal{M}_d$ is Ricci flat (it is also non-compact with infinite volume). If $\mathcal{Y}_{d-1}$ is a unit $(d-1)$-sphere $\mathbf{S}^{d-1}$, then the manifold $\mathcal{M}_d$ is flat. However, in all other cases $\mathcal{M}_d$ is *not* flat, in fact it has a conifold singularity at $r = 0$. For instance, in the $d = 6$ case if we take $\mathcal{Y}_5 = \mathbf{S}^5/\Gamma$ (where Γ is a subgroup of Spin(6)), then the space $\mathcal{M}_6$ is nothing but the orbifold $\mathbb{R}^6/\Gamma$. On the other hand, we can take $\mathcal{Y}_5$ to be other than $\mathbf{S}^5$ or an orbifold thereof. Then the resulting conifold is no longer an orbifold of $\mathbb{R}^6$. We will discuss an orientiworld model of this type in section 4.

[19] In the $d = 2$ case the manifold $\mathcal{M}_2$ is actually flat if the brane is tensionless. In particular, if we choose $\mathcal{M}_2$ to be a two dimensional "wedge" with a deficit angle, the singularity at the origin of the wedge is δ-function-like, and this solution corresponds to a *non-zero* tension brane with the tension proportional to the deficit angle. This solution was recently discussed in [32].

3.2 Linearized gravity

In this subsection we study gravity in the brane world solution discussed in the previous section. Thus, let us consider small fluctuations around the solution

$$G_{MN} = G^{(0)}_{MN} + h_{MN} \,, \tag{3.7}$$

where $G^{(0)}_{MN}$ is the background metric

$$G^{(0)}_{MN} = \begin{bmatrix} \eta_{\mu\nu} & 0 \\ 0 & \widetilde{G}_{ij} \end{bmatrix} . \tag{3.8}$$

The $(D-2)$-dimensional graviton $H_{\mu\nu} \equiv h_{\mu\nu}$ couples to the matter localized on the brane via

$$S_{\rm int} = \frac{1}{2}\int_\Sigma d^{D-d}x \; T_{\mu\nu} H^{\mu\nu} \,, \tag{3.9}$$

where $T_{\mu\nu}$ is the conserved energy-momentum tensor for the matter localized on the brane:

$$\partial^\mu T_{\mu\nu} = 0 \,. \tag{3.10}$$

The linearized equations of motion read

$$-\nabla^L\nabla_L h_{MN} + 2\nabla^L\nabla_{(M}h_{N)L} - \nabla_M\nabla_N h + G^{(0)}_{MN}\left(\nabla^L\nabla_L h - \nabla^L\nabla^K h_{LK}\right) = \\ = \frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\widetilde{T}_{MN}\delta^{(d)}(x^i - x^i_*) \,, \tag{3.11}$$

where the components of the "effective" energy-momentum tensor $\widetilde{T}_{MN}$ are given by

$$\begin{aligned} &\widetilde{T}_{\mu\nu} = T_{\mu\nu} - \widehat{M}_P^{D-d-2}\Big[-\partial^\lambda\partial_\lambda H_{\mu\nu} + 2\partial^\lambda\partial_{(\mu}H_{\nu)\lambda} - \partial_\mu\partial_\nu H + \eta_{\mu\nu}\left(\partial^\lambda\partial_\lambda H - \partial^\lambda\partial^\sigma H_{\lambda\sigma}\right)\Big] \,, \\ &\widetilde{T}_{\mu i} = \widetilde{T}_{ij} = 0 \,, \end{aligned} \tag{3.12}$$

and we have defined $h \equiv h^M{}_M$ and $H \equiv H^\mu{}_\mu$. Note that the fluctuations h_{MN} are assumed to vanish once we turn off the brane matter source $T_{\mu\nu}$.

Since the brane is tensionless, the D-dimensional diffeomorphisms are intact:

$$\delta h_{MN} = \nabla_M \xi_N + \nabla_N \xi_M \,. \tag{3.13}$$

We can, therefore, use the harmonic gauge:

$$\nabla^M h_{MN} = \frac{1}{2}\nabla_N h \,. \tag{3.14}$$

In the harmonic gauge the equations of motion simplify as follows:

$$-\nabla^L\nabla_L h_{MN} = \frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\left[\widetilde{T}_{MN} - \frac{1}{D-2}G^{(0)}_{MN}\widetilde{T}\right]\delta^{(d)}(x^i - x^i_*) \,, \tag{3.15}$$

where $T \equiv T_\mu{}^\mu$. Note that the graviphoton components $h_{\mu i}$ vanish — indeed, they do not couple to the conserved energy-momentum tensor on the brane. The graviscalar components $\chi_{ij} \equiv h_{ij}$ can only couple to the trace of the energy-momentum tensor T, which implies that

$$\chi_{ij} = \frac{1}{d}\widetilde{G}_{ij}\chi\,, \tag{3.16}$$

where $\chi \equiv \widetilde{G}^{ij}\chi_{ij}$. This gives:

$$\begin{aligned} -\nabla^L\nabla_L H_{\mu\nu} &= \frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\left[\widetilde{T}_{\mu\nu} - \frac{1}{D-2}\eta_{\mu\nu}\widetilde{T}\right]\delta^{(d)}(x^i - x^i_*)\,, \\ \nabla^L\nabla_L\chi &= \frac{d}{D-2}\frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\widetilde{T}\delta^{(d)}(x^i - x^i_*)\,. \end{aligned} \tag{3.17}$$

Note that the harmonic gauge implies that

$$\begin{aligned} \partial^\mu H_{\mu\nu} &= \frac{1}{2}\partial_\nu H + \frac{1}{2}\partial_\nu\chi\,, \\ \partial_j H &= -\frac{d-2}{d}\partial_j\chi\,. \end{aligned} \tag{3.18}$$

In particular, taking into account the physically required behavior at infinity, the last equation gives:

$$H = -\frac{d-2}{d}\chi\,. \tag{3.19}$$

We, therefore, have

$$\widetilde{T}_{\mu\nu} = T_{\mu\nu} - \widehat{M}_P^{D-d-2}\Big[-\partial^\lambda\partial_\lambda H_{\mu\nu} + \partial_\mu\partial_\nu\chi - \frac{d-1}{d}\eta_{\mu\nu}\partial^\lambda\partial_\lambda\chi\Big]\,. \tag{3.20}$$

As we will see, the presence of the term proportional to $\partial_\mu\partial_\nu\chi$ in $\widetilde{T}_{\mu\nu}$ has important implications.[20]

Let us Fourier transform the coordinates x^μ on the brane. Let the corresponding momenta be p^μ, and let $p^2 \equiv p^\mu p_\mu$. Then we have:

$$\begin{aligned} -\left(\nabla^i\nabla_i - p^2\right)H_{\mu\nu} &= \frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\left[\widetilde{T}_{\mu\nu}(p) - \frac{1}{D-2}\eta_{\mu\nu}\widetilde{T}(p)\right]\delta^{(d)}(x^i - x^i_*)\,, \\ \left[\nabla^i\nabla_i - p^2\right]\chi &= \frac{d}{D-2}\frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\widetilde{T}(p)\delta^{(d)}(x^i - x^i_*)\,, \end{aligned} \tag{3.21}$$

where

$$\widetilde{T}_{\mu\nu}(p) = T_{\mu\nu}(p) - \widehat{M}_P^{D-d-2}\Big[p^2 H_{\mu\nu} - p_\mu p_\nu\chi + \frac{d-1}{d}p^2\eta_{\mu\nu}\chi\Big]\,. \tag{3.22}$$

Let us now solve these equations following [4].

[20] The importance of this term was originally pointed out in [32].

Let us first consider the $p^2 \neq 0$ modes. Then, due to the fact that the d-dimensional propagator is divergent at the origin for $d \geq 2$, we have:

$$H_{\mu\nu} = \chi = 0\,, \qquad r \neq 0\,, \tag{3.23}$$

while on the brane we have

$$\begin{aligned} H_{\mu\nu}(p_\lambda, r=0) &= \frac{\widehat{M}_P^{2+d-D}}{p^2}\left[T_{\mu\nu}(p) - \frac{T(p)}{D-d-2}\left(\eta_{\mu\nu} - \frac{d}{d-1}\frac{p_\mu p_\nu}{p^2}\right)\right], \\ \chi(p_\lambda, r=0) &= \frac{d}{(d-1)(D-d-2)}\,\frac{\widehat{M}_P^{2+d-D}}{p^2}\,T(p)\,. \end{aligned} \tag{3.24}$$

Note that both the momentum as well as the tensor structures in the graviton propagator are $(D-d)$-dimensional. In particular, the $p^2 \neq 0$ modes are completely localized on the brane.

Next, let us study the $p^2 = 0$ modes:

$$\begin{aligned} -\nabla^i\nabla_i H_{\mu\nu} &= \frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}\left[T_{\mu\nu}(p) - \frac{1}{D-2}\eta_{\mu\nu}T(p) + \widehat{M}_P^{D-d-2}p_\mu p_\nu\chi\right]\delta^{(d)}(x^i - x^i_*)\,, \\ \nabla^i\nabla_i\chi &= \frac{d}{D-2}\frac{M_P^{2-D}}{\sqrt{\widetilde{G}}}T(p)\delta^{(d)}(x^i - x^i_*)\,. \end{aligned} \tag{3.25}$$

It is important to note that, if the brane matter is not conformal, then the solution for χ blows up at the location of the brane $x^i = x^i_*$, so that the equation for $H_{\mu\nu}$ is ill-defined due to the term proportional to $p_\mu p_\nu \chi$. Note that this term *cannot* be removed by a gauge transformation.[21] The aforementioned singularity is present even if the the manifold $\mathcal{M}_d$ is flat: $\mathcal{M}_d = \mathbb{R}^d$. Indeed, in this case χ diverges as $\sim |x - x_*|^{2-d}$ for $d > 2$, and as $\sim \ln(|x - x_*|)$ for $d = 2$. The same is true for a general smooth $\mathcal{M}_d$. In the case of conifolds we also have a similar singularity.

There are two points that arise due to the aforementioned singularity. First, it is clear that the linearized approximation breaks down. However, the second point might be relevant for addressing the former. Thus, we expect that ultra-violet physics should smooth out singularities of this type. One way this can happen is if the brane is not strictly δ-function-like, but has finite width. Let us discuss this possibility in a bit more detail. For simplicity we will focus on the case where the manifold $\mathcal{M}_d$ is flat: $\mathcal{M}_d = \mathbb{R}^d$.

3.3 A non-zero thickness brane

Smoothing out the brane in the presence of gravity is non-trivial. Thus, as was originally pointed out in [33], not only do we expect the Einstein-Hilbert term to

[21] Note that this term does not affect the coupling of the graviton $H_{\mu\nu}$ to the brane matter as $p^\mu T_{\mu\nu}(p) = 0$ for such matter. However, this term can be probed by bulk matter as $p^\mu T^{\rm bulk}_{\mu\nu}(p)$ need *not* be zero.

be generated on the brane, but also other terms, in particular, those involving the graviscalar as well as graviphoton components. Moreover, we actually expect that an infinite series of non-local terms will be present on the brane. Various non-trivial issues arise in this context, which are outside of the scope of this paper, and will be discussed elsewhere [34]. However, a simple prescription for smoothing out the brane, which we will discuss in a moment, will suffice for our purposes here.[22]

Thus, we will simply replace the δ-function-like brane Σ, which is a point-like object in $\mathcal{M}_d$, by an extended object $\widetilde{\Sigma}$ in $\mathcal{M}_d$. The corresponding action now reads:

$$\widetilde{S} = \widehat{M}_P^{D-d-2} \int_{\widetilde{\Sigma}} d^D x \; f^{(d)}(x^i - x^i_*) \sqrt{-\widehat{G}}\, \widehat{R} + M_P^{D-2} \int d^D x \sqrt{-G}\, R \,. \qquad (3.26)$$

Here $f^{(d)}(x^i - x^i_*)$ is a d-dimensional distribution, which replaces the d-dimensional δ-function $\delta^{(d)}(x^i - x^i_*)$. The $(D-d)$-dimensional Ricci scalar $\widehat{R}$ is constructed from the $(D-d)$-dimensional metric $\widehat{G}_{\mu\nu} = G_{\mu\nu}(x^\sigma, x^i)$, where the extra d-dimensional coordinates x^i are treated as parameters (in particular, note that $\widehat{R}$ does *not* contain derivatives w.r.t. x^i). Similarly, the coupling to brane matter is given by the term

$$S_{\rm int} = \frac{1}{2} \int_{\widetilde{\Sigma}} d^D x \, f^{(d)}(x^i - x^i_*)\, T_{\mu\nu} H^{\mu\nu} \,, \qquad (3.27)$$

where the energy-momentum tensor for the brane matter is conserved in the $(D-d)$-dimensional sense

$$\partial^\mu T_{\mu\nu} = 0 \,, \qquad (3.28)$$

and is *independent* of the coordinates x^i.

A straightforward guess for smoothing out the brane would be to choose $\widetilde{\Sigma}$ to be a d-dimensional object in $\mathcal{M}_d$. This is equivalent to replacing the d-dimensional δ-function $\delta^{(d)}(x^i - x^i_*)$ by a smooth distribution $f^{(d)}(x^i - x^i_*)$. In particular, we could consider a radially symmetric smooth distribution: $f^{(d)}(x^i - x^i_*) = f_1(r)$, where $r \equiv |x - x_*|$. A simple example of such a distribution would be (here $\theta(z)$ is the Heavyside step-function)

$$f_1(r) = \frac{1}{v_d \epsilon^d} \, \theta(\epsilon - r) \,, \qquad (3.29)$$

where v_d is the volume of a d-dimensional ball $\mathbf{B}_d$ with unit radius. Here ϵ plays the role of the brane thickness. In this case $\widetilde{\Sigma}$ is a d-dimensional ball of radius ϵ in $\mathcal{M}_d$. However, it is not difficult to show that, whatever our choice for the smooth d-dimensional distribution $f^{(d)}(x^i - x^i_*)$, the resulting system suffers from the presence of an *infinite* tower of *tachyonic* modes. In fact, these modes correspond to the

[22]One might expect that the aforementioned additional terms are suppressed by the brane thickness. However, ignoring at least some of such terms might not be justified due to the singularities arising in the δ-function limit. In fact, in the following we will argue that it appears to be necessary to include some of such terms for consistency.

"radion" χ. This can intuitively be understood from the fact that χ (as well as the trace of $H_{\mu\nu}$) couple to the interior (in $\mathcal{M}_d$) of such a smooth brane with the sign opposite to that of the traceless part of $H_{\mu\nu}$. In appendix B we illustrate this in the case of the distribution $f_1(r)$ given by (3.29).

This implies that $\widetilde{\Sigma}$ cannot be a d-dimensional object in $\mathcal{M}_d$. On the other hand, its dimension $\widetilde{d}$ in $\mathcal{M}_d$ should not be smaller than $(d-1)$ — indeed, if $\widetilde{d} \leq (d-2)$, we still expect singularities of the type discussed in the previous subsection to be present. This suggests that $\widetilde{\Sigma}$ should be an object of dimension $\widetilde{d} = (d-1)$ in $\mathcal{M}_d$. In fact, in this case we expect no singularities to be present. This is suggested by the fact that there are no singularities in the case of a codimension-1 brane in infinite-volume extra space [3].

We will therefore replace the d-dimensional δ-function $\delta^{(d)}(x^i - x^i_*)$ by a *singular* distribution such that it is smooth in $(d-1)$ dimensions, and has a δ-function-like singularity in the remaining one dimension. A simple example of such a distributions, which we will adopt in the following, is given by:

$$f(r) = \frac{1}{a_{d-1}\epsilon^{d-1}}\ \delta(r-\epsilon)\,, \tag{3.30}$$

where a_{d-1} is the area of a $(d-1)$-sphere $\mathbf{S}^{d-1}$ with unit radius. In this case $\widetilde{\Sigma}$ is a $(d-1)$-sphere of radius ϵ in $\mathcal{M}_d$. As we will see in the following, such partial smoothing out of the brane gives a consistent model in the present context.

Thus, we can rewrite our action (3.26) as follows:

$$\widetilde{S} = \widehat{M}_P^{D-d-2}\int d^Dx\, f(r)\sqrt{-\widehat{G}}\ \widehat{R} + M_P^{D-2}\int d^Dx\sqrt{-G}\,R\,, \tag{3.31}$$

and the coupling to brane matter is given by:

$$S_{\rm int} = \frac{1}{2}\int d^Dx\ f(r)\ T_{\mu\nu}H^{\mu\nu}\,. \tag{3.32}$$

The background solution is unaffected, and we can still choose it so that the entire D-dimensional space is flat. The linearized equations of motion are given by (3.11) with the d-dimensional δ-function $\delta^{(d)}(x^i - x^i_*)$ replaced by $f(r)$. It is straightforward to solve these equations (as before, the graviphoton components are vanishing, while the graviscalar components obey (3.16)):

$$\begin{aligned}\overline{H}_{\mu\nu} \equiv H_{\mu\nu} - \frac{1}{D-d}\eta_{\mu\nu}H = -M_P^{2-D}\Bigg[&\left(T_{\mu\nu}(p) - \frac{1}{D-d}\eta_{\mu\nu}T(p)\right)\Phi + \\ &+ \frac{d}{D-2}L^d\left(p_\mu p_\nu - \frac{1}{D-d}\eta_{\mu\nu}p^2\right)T(p)\Sigma\Bigg]\,,\\ H = -\frac{d-2}{d}\chi = -\frac{d-2}{D-2}M_P^{2-D}T(p)\widetilde{\Phi}\,.&\end{aligned} \tag{3.33}$$

Here Φ, Σ and $\widetilde{\Phi}$ are the solutions to the following equations

$$\begin{aligned}
\left(\partial^i\partial_i - p^2\right)\Phi &= \left[1 + L^d p^2 \Phi\right] f(r)\,, \\
\left(\partial^i\partial_i - p^2\right)\Sigma &= \left[\widetilde{\Phi} + L^d p^2 \Sigma\right] f(r)\,, \\
\left(\partial^i\partial_i - p^2\right)\widetilde{\Phi} &= \left[1 - \kappa L^d p^2 \widetilde{\Phi}\right] f(r)
\end{aligned} \tag{3.34}$$

subject to the condition that Φ, Σ and $\widetilde{\Phi}$ decay to zero away from the brane (in the following we will focus on the cases $d > 2$, where this is true even for the $p^2 = 0$ modes). Here we have introduced the notation

$$\kappa \equiv \frac{(d-1)(D-d-2)}{D-2} \tag{3.35}$$

to simplify expressions containing $\widetilde{\Phi}$.

With the choice (3.30) for the distribution function $f(r)$, we can solve for Φ, Σ and $\widetilde{\Phi}$ in terms of elementary functions for $d = 3$. Since in higher d the qualitative conclusions are the same as in $d = 3$, let us focus on this case. In appendix C we treat the general $d \geq 2$ case.

The $d = 3$ case. To solve for Φ, Σ and $\widetilde{\Phi}$, it is convenient to Wick rotate to the euclidean space. Let $p \equiv \sqrt{p^2}$. We then have the following radially symmetric solution for Φ:

$$\begin{aligned}
\Phi(r) &= -\frac{\epsilon}{L^d p^2 \sinh(p\epsilon) + a_d \epsilon^{d-1} p \exp(p\epsilon)} \, \frac{\sinh(pr)}{r}\,, \qquad r \leq \epsilon\,, \\
\Phi(r) &= -\frac{\epsilon \sinh(p\epsilon)}{L^d p^2 \sinh(p\epsilon) + a_d \epsilon^{d-1} p \exp(p\epsilon)} \, \frac{\exp\left(-p[r-\epsilon]\right)}{r}\,, \qquad r \geq \epsilon\,.
\end{aligned} \tag{3.36}$$

Next, the radially symmetric solution for Σ is given by:

$$\Sigma(r) = \widetilde{\Phi}(\epsilon)\,\Phi(r)\,. \tag{3.37}$$

Finally, the radially symmetric solution for $\widetilde{\Phi}$ is given by:

$$\begin{aligned}
\widetilde{\Phi}(r) &= \frac{\epsilon}{\kappa L^d p^2 \sinh(p\epsilon) - a_d \epsilon^{d-1} p \exp(p\epsilon)} \, \frac{\sinh(pr)}{r}\,, \qquad r \leq \epsilon\,, \\
\widetilde{\Phi}(r) &= \frac{\epsilon \sinh(p\epsilon)}{\kappa L^d p^2 \sinh(p\epsilon) - a_d \epsilon^{d-1} p \exp(p\epsilon)} \, \frac{\exp\left(-p[r-\epsilon]\right)}{r}\,, \qquad r \geq \epsilon\,.
\end{aligned} \tag{3.38}$$

Unlike the solution for $\Phi(r)$, the solution for $\widetilde{\Phi}(r)$ has a pole. Here we would like to discuss this point in a bit more detail.

Thus, the pole in $\widetilde{\Phi}(r)$ is located at $p = p_*$, where

$$p_* \left[1 - \exp(-2p_*\epsilon)\right] = \frac{2a_{d-1}\epsilon^{d-1}}{\kappa L^d}\,. \tag{3.39}$$

Suppose the brane thickness ϵ is small: $\epsilon \ll L$. Then we have:

$$p_*^2 \approx \frac{a_{d-1}\epsilon^{d-2}}{\kappa L^d}\,. \tag{3.40}$$

As we will see in the following, p_* is of order of the cross-over scale p_c above which gravity is $(D-d)$-dimensional, while below this scale it is expected to become D-dimensional. Thus, around the cross-over scale we have a single pole in the radion propagator (and, consequently, in the graviton propagator). In fact, this pole is *tachyonic*. Thus, consider the equation of motion for χ in the absence of the brane matter:

$$\left(\partial^i\partial_i - p^2\right)\chi = -\kappa L^d p^2 \chi f(r)\,. \tag{3.41}$$

Let us work with Minkowski momenta p^μ. We have a continuum of modes with $p^2 \leq 0$. These are massless and massive modes. However, we also have an isolated quadratically normalizable tachyonic mode with mass squared $m_*^2 = -p_*^2$, which is given by:

$$\begin{aligned} \chi_*(r) &= C\frac{\sinh(p_* r)}{r}\,, \qquad r \leq \epsilon\,,\\ \chi_*(r) &= C\sinh(p_*\epsilon)\,\frac{\exp\left(-p_*[r-\epsilon]\right)}{r}\,, \qquad r > \epsilon\,, \end{aligned} \tag{3.42}$$

where C is a constant.

At first it might seem that the presence of a tachyonic mode implies an instability in the system. However, as we will now argue, our approximations *cannot* be trusted at momenta $p^2 \lesssim p_c^2$, where we have defined the *cross-over* scale

$$p_c^2 \equiv \frac{a_{d-1}\epsilon^{d-2}}{L^d}\,. \tag{3.43}$$

Indeed, as we have already mentioned, we have ignored an infinite series of *non-local* terms on the brane, which are expected to be generated once we smooth it out. In particular, we have ignored an infinite number of non-local terms containing powers of the inverse momentum squared (in the momentum space):

$$\frac{1}{(p^2)^n}\,. \tag{3.44}$$

These terms are expected to be suppressed by the brane thickness ϵ. In fact, the expansion parameter is expected to be p_c^2/p^2:

$$\left(\frac{p_c^2}{p^2}\right)^n\,. \tag{3.45}$$

For momenta $p^2 \gg p_c^2$ these terms can be safely ignored. However, in the infra-red, that is, for the momenta $p^2 \lesssim p_c^2$, an *infinite* number of such non-local terms becomes important. We may then expect that the aforementioned tachyonic instability is an artifact of dropping such non-local terms.[23]

Let us discuss this point in a bit more detail. Thus, consider the momenta $p^2 \ll 1/\epsilon^2$. Then we have:

$$\begin{aligned} \Phi(\epsilon) &\approx -\frac{1}{L^d}\,\frac{1}{p^2+p_c^2}\,, \\ \widetilde{\Phi}(\epsilon) &\approx \frac{1}{\kappa L^d}\,\frac{1}{p^2-p_*^2}\,, \\ \Sigma(\epsilon) &\approx -\frac{1}{\kappa L^{2d}}\,\frac{1}{p^2+p_c^2}\,\frac{1}{p^2-p_*^2}\,, \end{aligned} \tag{3.46}$$

where $p_*^2 \approx p_c^2/\kappa$. On the brane we therefore have:

$$\begin{aligned} H_{\mu\nu}(r=\epsilon) \approx \frac{\widehat{M}_P^{2+d-D}}{p^2+p_c^2}\Bigg[T_{\mu\nu}(p) - \frac{1}{D-d-2}\eta_{\mu\nu}T(p) + \qquad\qquad (3.47)\\ +\frac{d}{(d-1)(D-d-2)}\left(p_\mu p_\nu - \eta_{\mu\nu}\frac{(d-2)p_c^2+2(d-1)p_*^2}{d(D-d)}\right)\frac{T(p)}{p^2-p_*^2}\Bigg]\,, \end{aligned}$$

$$\chi(r=\epsilon) \approx \frac{d}{(d-1)(D-d-2)}\,\frac{\widehat{M}_P^{2+d-D}}{p^2-p_*^2}\,T(p)\,. \tag{3.48}$$

As we have already mentioned, we can trust our approximations for the momenta $p_c^2 \ll p^2 \ll 1/\epsilon^2$. Then on the brane we have:

$$\begin{aligned} H_{\mu\nu}(r=\epsilon) &\approx \frac{\widehat{M}_P^{2+d-D}}{p^2}\left[T_{\mu\nu}(p) - \frac{T(p)}{D-d-2}\left(\eta_{\mu\nu} - \frac{d}{d-1}\,\frac{p_\mu p_\nu}{p^2}\right)\right]\,, \\ \chi(r=\epsilon) &\approx \frac{d}{(d-1)(D-d-2)}\,\frac{\widehat{M}_P^{2+d-D}}{p^2}\,T(p)\,. \end{aligned} \tag{3.49}$$

This shows that both the momentum as well as the tensor structures of the graviton propagator on the brane are indeed $(D-d)$-dimensional. On the other hand, for the momenta $p^2 \lesssim p_c^2$ the above approximation is expected to break down. In fact, we can get a hint of what kind of non-local terms would be relevant for, in particular, removing the tachyonic state by expanding (3.47) and (3.48) in powers of p_c^2/p^2. Finally, note that the transverse-traceless components of $H_{\mu\nu}$ in (3.47) behave as those of a *massive* $(D-d)$-dimensional propagator with mass squared equal p_c^2.

3.4 Non-zero tension solutions

For the following discussions it will be useful to understand the non-zero tension solutions, which we would like to discuss in this subsection. Thus, consider the

[23] Here we note that, had we chosen a smooth d-dimensional distribution such as (3.29), we would have had an infinite tower of such tachyonic modes (see appendix B for details). In this case it is less clear how the aforementioned non-local terms could remove all the tachyonic states.

following model:

$$\widetilde{S} = \widehat{M}_P^{D-d-2} \int d^{D-d}x \ f(r) \ \sqrt{-\widehat{G}} \left[\widehat{R} - \widehat{\Lambda}\right] + M_P^{D-2} \int d^D x \ \sqrt{-G} \ R \, . \tag{3.50}$$

Here

$$\tau \equiv \widehat{M}_P^{D-d-2} \widehat{\Lambda} \tag{3.51}$$

plays the role of the brane tension. That is, in the δ-function limit τ is the tension of the brane Σ, which is a point in the extra space $\mathcal{M}_d$.

The equations of motion read

$$R_{MN} - \frac{1}{2} G_{MN} R + \frac{\sqrt{-\widehat{G}}}{\sqrt{-G}} \delta_M{}^\mu \delta_N{}^\nu \left[\widehat{R}_{\mu\nu} - \frac{1}{2} \widehat{G}_{\mu\nu} \left(\widehat{R} - \widehat{\Lambda}\right)\right] L^d f(r) = 0 \, . \tag{3.52}$$

To solve these equations, let us use the following ansätz for the background metric:

$$ds^2 = \exp(2A) \ \eta_{\mu\nu} dx^\mu dx^\nu + \exp(2B) \ \delta_{ij} dx^i dx^j \, , \tag{3.53}$$

where A and B are functions of x^i but are independent of x^μ. We then have:

$$R_{\mu\nu} = -\eta_{\mu\nu} e^{2(A-B)} \left[\partial^i \partial_i A + (D-d) \partial^i A \partial_i A + (d-2) \partial^i A \partial_i B\right] \, , \tag{3.54}$$

$$\begin{aligned} R_{ij} = & (d-2) \left[\partial_i B \partial_j B - \partial_i \partial_j B\right] + (D-d) \left[\partial_i A \partial_j B + \partial_j A \partial_i B - \partial_i A \partial_j A - \partial_i \partial_j A\right] - \\ & - \delta_{ij} \left[\partial^k \partial_k B + (d-2) \partial^k B \partial_k B + (D-d) \partial^k A \partial_k B\right] \, , \end{aligned} \tag{3.55}$$

$$\begin{aligned} R = & -e^{-2B} \Big[2(d-1) \partial^i \partial_i B + 2(D-d) \partial^i \partial_i A + (d-1)(d-2) \partial^i B \partial_i B + \\ & + (D-d)(D-d+1) \partial^i A \partial_i A + 2(D-d)(d-2) \partial^i A \partial_i B\Big] \, . \end{aligned} \tag{3.56}$$

Here we are interested in radially symmetric solutions where A and B are functions of r. The equations of motion then read (here prime denotes derivative w.r.t. r):

$$(d-2) \left[(B')^2 - B'' + \frac{1}{r} B'\right] + (D-d) \left[2A'B' - (A')^2 - A'' + \frac{1}{r} A'\right] = 0 \, , \tag{3.57}$$

$$\begin{aligned} & (d-2) \left[B'' + \frac{1}{2}(d-3)(B')^2 + (d-2)\frac{1}{r} B'\right] + \\ & + (D-d) \left[A'' + \frac{1}{2}(D-d+1)(A')^2 + (d-2)\frac{1}{r} A' + (d-3) A'B'\right] = 0 \, , \end{aligned} \tag{3.58}$$

$$\begin{aligned} & (D-d-1) \left[A'' + \frac{1}{2}(D-d)(A')^2 + (d-1)\frac{1}{r} A' + (d-2) A'B'\right] + \\ & + (d-1) \left[B'' + \frac{1}{2}(d-2)(B')^2 + (d-1)\frac{1}{r} B'\right] + \frac{1}{2} e^{(2-d)B} \widehat{\tau} f(r) = 0 \, , \end{aligned} \tag{3.59}$$

where $\widehat{\tau} \equiv \tau / M_P^{D-2}$.

Here we are interested in solutions with *infinite* volume extra dimensions.[24] In $d = 2$ there is a simple solution of this type with $A \equiv 0$. This solution was studied in [32]. Here we will focus on the $d > 2$ cases. We can write down approximate solutions for small brane tension. Thus, let

$$\widehat{\tau} \equiv \gamma \epsilon^{d-2} \,, \tag{3.60}$$

where the dimensionless parameter γ is small: $\gamma \ll 1$. Then it is not difficult to see that we can drop terms non-linear in A' and B' (assuming that we are looking for solutions non-singular at $r = 0$ such that A and B asymptote to constant values at $r \to \infty$). In particular, the corrections due to the non-linear terms in this case are suppressed by powers of γ. The above equations then simplify as follows:

$$\begin{aligned} B' &= -\frac{D-d}{d-2} A' \,, \\ A'' + (d-1)\frac{1}{r} A' &= \frac{d-2}{D-2} \frac{\gamma}{2 a_{d-1} \epsilon} e^{(2-d)B} \delta(r - \epsilon) \,. \end{aligned} \tag{3.61}$$

The solution is given by (here we have fixed the integration constants so that both A and B vanish on the brane):

$$\begin{aligned} A(r) &= \frac{\gamma}{2(D-2) a_{d-1}} \left[1 - \frac{\epsilon^{d-2}}{r^{d-2}} \right] \theta(r - \epsilon) \,, \\ B(r) &= -\frac{D-d}{d-2} A(r) \,. \end{aligned} \tag{3.62}$$

Note that A and B are constant for $r < \epsilon$. In particular, these solutions are non-singular, and the D-dimensional space-time becomes flat as $r \to \infty$.

We would like to end this subsection with a few remarks. To find exact solutions, one can proceed as follows. Consider the sum of (3.57) and (3.58):

$$(d-2) \left[(B')^2 + \frac{2}{r} B' \right] + (D-d) \left[\frac{D-d-1}{d-1} (A')^2 + \frac{2}{r} A' + 2 A' B' \right] = 0 \,. \tag{3.63}$$

Note that this equation does not contain second derivatives of A and B. The solution for B' is given by:

$$B' = -\frac{1}{r} - \frac{D-d}{d-2} A' \pm \sqrt{\frac{1}{r^2} + \frac{(D-d)(D-2)}{(d-1)(d-2)^2} (A')^2} \,. \tag{3.64}$$

To obtain solutions with infinite-volume extra space we should choose the plus root in the above solution. The non-singular solutions that asymptote to a flat space then

[24] For any d there exists a simple solution with $A \equiv 0$. However, in this solution the extra space has finite volume for $d > 2$.

have the property that A and B are constant for $r < \epsilon$. That is, the space inside of the $(d-1)$-sphere of radius ϵ is actually flat. This will be important in the following.

Finally, let us note that in $d = 2$ we have substantial simplifications, and it is not difficult to solve the equations for A and B exactly. In particular, as was recently discussed in [32], in $d = 2$ we have a solution where the D-dimensional space-time is a product of the $(D-2)$-dimensional Minkowski space-time and a 2-dimensional extra space with a deficit angle (in the δ-function limit this is a 2-dimensional "wedge" with a conical singularity at the origin, while in the case of a smoothed out brane the 2-dimensional extra space is non-singular). In this example, which we discuss in appendix D, one can explicitly see that, in the appropriate regime, we reproduce $(D-2)$-dimensional gravity on the brane even for non-vanishing brane tension.[25]

3.5 Brane thickness in orientiworld

Up to various subtleties mentioned in the beginning of subsection 3.3, following [4] we can expect that, above the cross-over scale[26]

$$r_c \sim \sqrt{\frac{\widehat{M}_P^{D-d-2}}{M_P^{D-2}}\,\frac{1}{\epsilon^{d-2}}} \tag{3.65}$$

all the way down to the scales of order of the brane thickness ϵ, gravity on the brane is $(D-d)$-dimensional.

In the orientiworld context we have $D = 10$, $d = 6$, and (g_s is the string coupling):

$$M_P^8 \sim M_s^8/g_s^2\,. \tag{3.66}$$

We, therefore, have:

$$r_c \sim g_s\,\frac{\widehat{M}_P}{M_s^2}\,\frac{1}{M_s^2\epsilon^2}\,. \tag{3.67}$$

Phenomenologically we must require that $\widehat{M}_P$ is the four-dimensional Planck scale: $\widehat{M}_P \sim 10^{18}\,\mathrm{GeV}$. Let us also require that the cross-over scale is at least as large as the present size of the observable Universe, that is, the Hubble size $r_H \sim 10^{28}\,\mathrm{cm}$. Then $r_c\widehat{M}_P \gtrsim 10^{60}$. We will also assume that the string coupling $g_s \sim 1$. Then we have the following upper bound on the brane thickness:

$$\epsilon \lesssim 10^{-30}\,\frac{\widehat{M}_P}{M_s^2}\,. \tag{3.68}$$

The phenomenological lower bound on M_s is $M_s \gtrsim \mathrm{TeV}$. This then implies that the lower bound on the brane thickness is

$$\epsilon \lesssim \frac{1}{\widehat{M}_P}\,. \tag{3.69}$$

[25] A similar analysis in the $d > 2$ cases will be presented elsewhere [35].

[26] This formula is correct for $d > 2$. For $d = 2$ we have a logarithm of ϵ in the expression for r_c. See appendix C for details.

This lower bound is saturated when

$$\epsilon \sim \frac{1}{\widehat{M}_P}: \qquad M_s \sim \text{TeV}\,, \qquad r_c \sim r_H\,. \tag{3.70}$$

If M_s is much larger than TeV and/or if r_c is much larger than r_H, then ϵ must be much smaller than the inverse four-dimensional Planck scale.

If ϵ is non-zero, we expect that $1/\epsilon$ corresponds to some threshold scale in the theory. In particular, this ultra-violet threshold must be responsible for smoothing out the aforementioned singularities. It is not unreasonable to expect that the ultra-violet physics that is responsible for smoothing out these singularities should not be unrelated to the ultra-violet physics that is responsible for generating the four-dimensional Einstein-Hilbert term.[27] If so, then the aforementioned bound should be saturated, and we have (3.70). That is, the string scale is around TeV, and gravity becomes higher dimensional at the present Hubble size. Clearly, such a scenario would have very important experimental implications. In particular, the string excitations could be observable at LHC. On the other hand, the fact that gravity undergoes a transition from being 4-dimensional to being higher dimensional would have important cosmological implications. For instance, one might attempt to use this to explain an accelerated Universe with vanishing 4-dimensional cosmological constant as in [36].

Now we come to the key question. What is the ultra-violet physics responsible for generating the four-dimensional Einstein Hilbert term and smoothing out the brane? Moreover, how can the corresponding threshold be so much higher than the string scale? The next section is devoted to discussing these issues.

4. Four-dimensional Einstein-Hilbert term in orientiworld

Let us understand the origin of the four-dimensional Einstein-Hilbert term in the orientiworld context. If we have a non-conformal field theory on D3-branes, and the field theory description is adequate up to some threshold scale Λ, which we will refer to as the ultra-violet cut-off scale, then from the four-dimensional effective field theory viewpoint we expect that

$$\widehat{M}_P^2 \sim \Lambda^2\,. \tag{4.1}$$

Thus, we must understand what Λ is in a given orientiworld setup. To do this, we will start from a simple $\mathcal{N}=2$ supersymmetric model, and then move onto non-conformal $\mathcal{N}=1$ models discussed in subsection 2.4.

[27] Otherwise we would have to have three different threshold scales in the theory: $M_s \ll \widehat{M}_P \ll 1/\epsilon$. Note that we could assume that $M_s \sim \widehat{M}_P$. Then we would have to have an ultra-high threshold scale $1/\epsilon \gtrsim 10^{30}\widehat{M}_P$.

4.1 A simple $\mathcal{N} = 2$ example

As we discussed in subsection 2.3, non-compact $\mathcal{N} = 2$ orientiworld models are all conformal. Here we would like to discuss a non-conformal model. This can be done in a setup where two of the six extra dimensions are compact.

Thus, consider type IIB on $T^2 \times (\mathbb{C}^2/\Gamma)$. For simplicity we will take a square 2-torus T^2:

$$x^i \sim x^i + 2\pi r\,, \qquad i = 1, 2\,. \tag{4.2}$$

We will assume that $r^2 \gg \alpha'$. The orbifold group Γ can be an appropriate subgroup of SU(2). Here we would like to discuss orientifolds of such backgrounds containing O3-planes. For simplicity we will focus on the cases where we have no O7-planes. Then $\Gamma = \mathbb{Z}_M$, where M is odd. The choice of M is not going to be critical here, so for illustrative purposes we will take the simplest example: $\Gamma = \mathbb{Z}_3 = \{1, \theta, \theta^2\}$. The twist θ acts as follows on the complex coordinates z_2, z_3 on $\mathbb{C}^2$:

$$\theta : z_2 \to \omega z_2\,, \qquad \theta : z_3 \to \omega^{-1} z_3\,, \tag{4.3}$$

where $\omega \equiv \exp(2\pi i/3)$.

Now consider the ΩJ orientifold of type IIB on $T^2 \times (\mathbb{C}^2/\Gamma)$, where J acts on the complex coordinates z_1, z_2, z_3 as follows ($z_1 \equiv x^1 + ix^2$ is the complex coordinate on T^2):

$$J : z_\alpha \to -z_\alpha\,, \qquad \alpha = 1, 2, 3\,. \tag{4.4}$$

Note that J has four fixed points in $T^2 \times (\mathbb{C}^2/\Gamma)$. These are located at $z_2 = z_3 = 0$ and

$$(x^1, x^2) = (0, 0)\,, \qquad (\pi r, 0)\,, \qquad (0, \pi r)\,, \qquad (\pi r, \pi r)\,. \tag{4.5}$$

We will place an O3$^-$-plane at each of these points.

The twisted closed string states have non-trivial couplings to the orientifold planes, which gives rise to tadpoles. To cancel the tadpoles we must introduce D3-branes. The number of these D3-branes is arbitrary. We can cancel tadpoles *locally* at each O3$^-$-plane if we put $(3n + 2)$ D3-branes on top of each of the O3$^-$-planes and choose the twisted Chan-Paton matrices as follows [6]:

$$\gamma_\theta^I = \mathrm{diag}(\omega I_n, \omega^{-1} I_n, I_{n+2})\,, \tag{4.6}$$

where I labels the four O3$^-$-planes. On each of the four sets of D3-branes we then have a *conformal*[28] $\mathcal{N} = 2$ gauge theory with the gauge group $\mathrm{SU}(n) \otimes \mathrm{SO}(n+2)$ and matter hypermultiplets transforming in $(\mathbf{A}, \mathbf{1})$ and $(\mathbf{n}, \mathbf{n+2})$ (we have dropped the U(1) factors for the reasons discussed in subsection 2.3), where $\mathbf{A}$ is the two-index anti-symmetric representation of SU(n) (whose dimension is $n(n-1)/2$).

[28]More precisely, these gauge theories are conformal at low energies — see below.

We can, however, obtain a non-conformal D3-brane gauge theory if we cancel tadpoles only *globally* and not locally. Thus, let us place $(3N+8)$ D3-branes on top of the O3$^-$-plane located at the origin of T^2, and place no D3-branes on top of the other three O3$^-$-planes. We can cancel the twisted tadpoles globally if we choose the twisted Chan-Paton matrix acting on the D3-branes as follows [6]:

$$\gamma_\theta = \mathrm{diag}(\omega I_N, \omega^{-1} I_N, I_{N+8})\,. \tag{4.7}$$

The D3-brane gauge theory is then a non-conformal $\mathcal{N}=2$ gauge theory with the gauge group $\mathrm{SU}(N)\otimes\mathrm{SO}(N+8)$ and matter hypermultiplets transforming in $(\mathbf{A},\mathbf{1})$ and $(\mathbf{N},\mathbf{N+8})$, where $\mathbf{A}$ is the two-index anti-symmetric representation of $\mathrm{SU}(N)$.

As we discussed in subsection 2.3, the massive string thresholds do not contribute into the four-dimensional gauge coupling renormalization. The four-dimensional field theory description is therefore adequate up the the open string *winding* threshold [17, 24, 25] (see appendix A for details), whose mass is given by r/α'. The four-dimensional gauge theory cut-off, therefore, is given by (up to a dimensionless numerical coefficient, which is subtraction scheme dependent):

$$\Lambda \sim \frac{r}{\alpha'}\,. \tag{4.8}$$

Our field theory intuition then tells us that the induced 4-dimensional Planck scale should be

$$\widehat{M}_P \sim \Lambda \sim \frac{r}{\alpha'}\,. \tag{4.9}$$

Let us now see this in the string theory language.

Note that the global tadpole cancellation guarantees that we have no *tree-channel* infra-red divergences. However, unless the twisted tadpoles are canceled locally, we do have tree-channel *ultra-violet* divergences (which correspond to the loop-channel infra-red divergences). These divergences are logarithmic in the renormalization of the gauge couplings for massless D3-brane gauge bosons. That is, the loop-channel ultra-violet and infra-red divergences in the gauge coupling renormalization are *not* identical in this background. This is possible because, even though the massive string excitations still do not contribute into the corresponding amplitudes, the massive open string winding states do as they *are* BPS states (this effectively introduces the ultra-violet cut-off Λ in the loop channel). In the tree channel this corresponds to non-vanishing contributions from massive Kaluza-Klein modes of twisted closed string states, which give rise to tree-channel ultra-violet divergences.

What about the graviton kinetic term on the D3-branes? Actually, the four-dimensional Einstein-Hilbert term receives contributions both from the open as well as *closed* string diagrams with two external graviton lines. Let us discuss this in more detail at the one-loop order. First consider the torus amplitude (this is the genus-1 amplitude with no boundaries or cross-caps). Massless as well as massive closed string states run in the loop. The untwisted closed string states renormalize the 10-dimensional part of the effective action. The twisted closed string states could *a priori* give rise to a *six-dimensional* Einstein-Hilbert term (among other terms) localized at the orbifold fixed point:

$$\overline{M}_P^4 \int d^6x \, \sqrt{-\overline{G}} \, \overline{R} \, . \tag{4.10}$$

However, the contribution to the 6-dimensional Planck scale $\overline{M}_P \sim M_s$ due to the twisted closed string states propagating in the closed string loop actually vanishes. This follows from the general result of [38], where it was shown that in type-II string theory on $\mathbb{R}^{1,3} \times \mathcal{M}_6$, where $\mathcal{M}_6$ is some (compact or non-compact) Ricci-flat manifold, the one-loop contribution to the four-dimensional Planck scale is proportional to the Euler characteristic of $\mathcal{M}_6$. In our case the latter vanishes, hence the above conclusion. This, in turn, can be related to the fact that before orientifolding in the closed string sector we have 16 supercharges.

Next, let us consider the Klein bottle contribution to $\overline{M}_P^4$. Let us first discuss contributions coming from the twisted closed string states running in the *loop channel.* Such contributions actually *vanish.* This is because only left-right symmetric states can contribute into the Klein bottle, while the twisted closed string states are *not* left-right symmetric. In particular, note that the orientifold projection Ω does not commute with the twist θ [37]:

$$\Omega\theta\Omega^{-1} = \theta^{-1} \, . \tag{4.11}$$

This, in particular, implies that the twisted closed string ground states are not left-right symmetric, so the twisted closed string states do not contribute to the Klein bottle amplitude. However, the untwisted closed string states running in the loop channel do contribute into $\overline{M}_P^4$ [39]. One way to see this is to go to the tree channel. In the tree channel these contributions correspond to both untwisted as well as twisted exchanges. The former only renormalize 10-dimensional terms. However, the latter give rise to a six-dimensional Einstein-Hilbert term (among other terms). The corresponding contribution to the six-dimensional Planck scale $\overline{M}_P^4$ is of order M_s^4. Upon compactification on T^2 this translates into the following contribution to the four-dimensional Planck scale $\widehat{M}_P^2$:

$$\overline{M}_P^4 (2\pi r)^2 \sim \frac{r^2}{(\alpha')^2} \, . \tag{4.12}$$

In particular, this is consistent with our field theory intuition.

Let us now discuss the open string one-loop amplitudes. An important point here is that we can attach the external graviton lines anywhere on the tube. In particular, there is nothing singular about attaching these lines to the boundaries or cross-caps.

Thus, if we do so, we do not change the Chan-Paton structure of the amplitude. In fact, the correct way of thinking about these amplitudes is that we consider all possible positions for the insertions, and then integrate over these positions. It is then clear that the untwisted annulus and Möbius strip amplitudes (that is, those with the untwisted Chan-Paton matrix γ_1 acting on a boundary) can only contribute to the renormalization of the 10-dimensional terms. On the other hand, the twisted annulus and Möbius strip amplitudes (that is, those with the twisted Chan-Paton matrix γ_θ or γ_{θ^2} acting on a boundary), can only contribute to the 6-dimensional Planck scale, and this contribution is of order M_s^4. In particular, there is no truly 4-dimensional Einstein-Hilbert term coming from these contributions. Since the coordinates x^i are compact, we once again obtain a contribution of order (4.12). That is, the 4-dimensional Planck scale is of order

$$\widehat{M}_P^2 \sim \frac{r^2}{(\alpha')^2}\,, \tag{4.13}$$

which is consistent with the field theory expectation (4.8).

Note that if we decompactify the torus ($r \to \infty$), the 4-dimensional Planck scale goes to *infinity*. The D3-brane field theory remains non-conformal in this case. This might appear to contradict our earlier claims that all non-compact $\mathcal{N} = 2$ models are conformal. Actually, there is no contradiction here. Indeed, even though we took the other three O3$^-$-planes to infinity, they do *not* decouple as the twisted closed string states that couple them to the D3-branes propagate in *two* extra dimensions, and the massless 2-dimensional euclidean propagator is logarithmically divergent at large distances. Since the tadpoles are not canceled locally in this background, the D3-brane theory becomes infinitely strongly coupled. One can see this from the effective field theory viewpoint by noting that, if we assume that the $\mathrm{SU}(N)$ and $\mathrm{SO}(N+8)$ gauge couplings are finite at any finite energy scale, the former blows up at some higher but finite scale (Landau pole) once we take the cut-off Λ to infinity (which is what happens when we take r to infinity).

Let us now go back to the case where we cancel all tadpoles locally. The low energy theory on each set of D3-branes is then conformal. Note, however, that at finite r we have a finite 4-dimensional Planck scale. Thus, we have a contribution from compactifying the 6-dimensional Einstein-Hilbert term, which comes from the closed string Klein bottle amplitude as well as the open string annulus and Möbius strip amplitudes, on T^2. In particular, even though the D3-brane low energy effective field theories are conformal, the open string annulus and Möbius strip amplitudes do contribute into the six-dimensional Planck scale as we have a tower of massive open string excitations.[29] Now we come to one of the key points of our discussion here — in the decompactification limit, even in the conformal case, the 4-dimensional

[29]Note that having a non-vanishing four-dimensional Planck scale upon compactification is *not* inconsistent with the fact that the low energy effective field theories on the branes are conformal.

Planck scale is not zero but *infinite*. This is due to the fact that, in these backgrounds it actually comes from the compactification of the *six-dimensional* Einstein-Hilbert term localized at the orbifold fixed point. Also, note that in the conformal case, once we decompactify the 2-torus, the O3$^-$-planes and D3-branes at the other three fixed points decouple from those at the origin. And gravity now decouples not because the 4-dimensional Planck scale goes to zero but because it goes to infinity.

Before we end this subsection, we would like to make the following remark. From the above discussion it might be tempting to consider a hybrid scenario with two extra dimensions (untouched by the orbifold) compactified on a large torus and four others non-compact. In particular, in this case we could take $M_s \sim$ TeV, and obtain the desired four-dimensional Planck scale $\widehat{M}_P \sim 10^{18}$ GeV by choosing the size r of the two compact extra dimensions to be of order of a millimeter as in [2]. However, to obtain the desired cross-over scale r_c, the brane thickness in this case would have to be extremely small. Thus, let us focus on distance scales above the compactification size r. Then we can integrate out the compact extra dimensions, which leaves us with a model equivalent to having a 3-brane in 8-dimensional bulk. The 4-dimensional and 8-dimensional Planck scales are given by

$$\begin{aligned} \widehat{M}_P^2 &\sim M_s^4 r^2 \,, \\ M_P^6 &\sim M_s^8 r^2 \,. \end{aligned} \tag{4.14}$$

Following our discussion in subsection 3.5, we then have the following relation between the cross-over scale r_c and the brane thickness ϵ:

$$r_c \sim \frac{1}{M_s^2 \epsilon} \,. \tag{4.15}$$

To have r_c around or above the present Hubble size we must require that ϵ is around or smaller than $10^{-30} \widehat{M}_P^{-1}$, which would require a presence of a new ultra-high threshold $1/\epsilon$ in the theory.

4.2 A conifold example

From our discussion in the previous subsection it follows that, if we consider an oriented background of type IIB on $\mathbb{C} \times (\mathbb{C}^2/\mathbb{Z}_2)$ in the presence of D3-branes, we have a *six-dimensional* Einstein-Hilbert term localized at the orbifold fixed point. The gauge theory on the D3-branes in this case is the $\mathcal{N}=2$ supersymmetric theory with the $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$ gauge group and matter hypermultiplets transforming in $(\mathbf{N}, \overline{\mathbf{N}})$ and $(\overline{\mathbf{N}}, \mathbf{N})$. Note that this theory is conformal. Moreover, there is no ultraviolet cut-off in the D3-brane gauge theory. This is because the string excitations do

Indeed, we do have massive open string winding states in this case, which determine the four-dimensional effective field theory cut-off Λ (as the massive open string excitations do not contribute into the four-dimensional gauge coupling renormalization).

not contribute into the gauge coupling renormalization as they are non-BPS states (and we have $\mathcal{N} = 2$ supersymmetry). That is, the ultra-violet cut-off Λ in the theory is actually infinite, so the four-dimensional Planck scale is also infinite.

Now we would like to deform this theory so that it is no longer conformal. This can be done as follows. Note that the six-dimensional manifold $\mathcal{M}_6$ transverse to the D3-branes is a Ricci-flat manifold with the metric

$$ds^2 = dr^2 + r^2 \gamma_{\alpha\beta} dy^\alpha dy^\beta \,, \tag{4.16}$$

where $\gamma_{\alpha\beta}$ is a metric on a compact 5-dimensional Einstein manifold $\mathcal{Y}_5 = \mathbf{S}^5/\Gamma$ ($\mathbf{S}^5$ is a unit 5-sphere, $\sum_{\alpha=1}^{3} |z_\alpha|^2 = 1$), $\Gamma = \mathbb{Z}_2$, and the action of the generator R of $\mathbb{Z}_2$ on $\mathbf{S}^5$ is given by

$$R z_1 = z_1 \,, \qquad R z_{2,3} = -z_{2,3} \,. \tag{4.17}$$

Note that the fixed point locus of R in $\mathbf{S}^5$ is a unit circle $\mathbf{S}^1$. We can deform the orbifold by blowing up this fixed circle. The resulting space $\widetilde{\mathcal{M}}_6$ is Ricci-flat with the metric

$$ds^2 = dr^2 + r^2 \widetilde{\gamma}_{\alpha\beta} dy^\alpha dy^\beta \,, \tag{4.18}$$

where $\widetilde{\gamma}_{\alpha\beta}$ is a metric on a compact 5-dimensional Einstein manifold $\mathcal{Y}_5 = T^{1,1} = (\mathrm{SU}(2) \times \mathrm{SU}(2))/\,\mathrm{U}(1)$ [40]. The blow-up breaks supersymmetry from $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. In particular, in the low energy action it corresponds to adding a mass term to the adjoint chiral $\mathcal{N} = 1$ supermultiplet in the $\mathcal{N} = 2$ vector multiplet. This mass term is odd under the interchange of the two $\mathrm{SU}(N)$ subgroups as the twisted closed string modes associated with the blow-up are odd under the $\mathbb{Z}_2$ twist. We therefore have for this mass term [40]:

$$\frac{m}{2} \left[\mathrm{Tr} \left(\Phi^2 \right) - \mathrm{Tr} \left(\widetilde{\Phi}^2 \right) \right] , \tag{4.19}$$

where Φ and $\widetilde{\Phi}$ are the aforementioned adjoint chiral $\mathcal{N} = 1$ supermultiplets corresponding to the first and the second $\mathrm{SU}(N)$ subgroups, respectively.

Let us decompose the original matter hypermultiplets in terms of the $\mathcal{N} = 1$ chiral supermultiplets:

$$A_a = 2 \times (\mathbf{N}, \overline{\mathbf{N}}) \,, \qquad B_a = 2 \times (\overline{\mathbf{N}}, \mathbf{N}) \,, \qquad a = 1, 2 \,. \tag{4.20}$$

In the original $\mathcal{N} = 2$ model we have the following superpotential (in the $\mathcal{N} = 1$ language):

$$\mathcal{W} = g \,\mathrm{Tr} \left[\Phi \left(A_1 B_1 + A_2 B_2 \right) \right] + g \,\mathrm{Tr} \left[\widetilde{\Phi} \left(B_1 A_1 + B_2 A_2 \right) \right] , \tag{4.21}$$

where g is the Yang-Mills gauge coupling. Integrating out the fields $\Phi, \widetilde{\Phi}$, we obtain the following effective superpotential

$$\mathcal{W}_{\mathrm{eff}} = \frac{g^2}{2m} \left[\mathrm{Tr}(A_1 B_1 A_2 B_2) - \mathrm{Tr}(B_1 A_1 B_2 A_2) \right] , \tag{4.22}$$

which is non-renormalizable. In the infra-red the $\mathcal{N}=1$ supersymmetric $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$ gauge theory with chiral matter supermultiplets A_a and B_a flows into a superconformal field theory [40], which is what we expect from the AdS/CFT correspondence [41, 42, 43, 44, 12, 5, 6, 40, 45]. The above superpotential at the infra-red fixed point becomes a marginal deformation giving rise to a line of fixed points [40].[30]

Even though the above blown-up theory is conformal in the infra-red, it is not a conformal theory. In particular, we have a massive threshold m in the theory. Note that at the energy scales much higher than m (that is, in the ultra-violet) the field theory once again becomes conformal — it is simply described by the original $\mathcal{N}=2$ superconformal field theory. However, since the blown-up theory is not conformal, as was originally pointed out in [29], we expect the Einstein-Hilbert term to be generated on the D3-branes. Here we would like to understand what the corresponding four-dimensional Planck scale should be.

To begin with, note that the blown-up space $\widetilde{\mathcal{M}}_6$ is a *conifold*, and the D3-branes are located at the conifold singularity. In particular, we no longer have an orbifold singularity. Let the blow-up size be ϵ. Then we have

$$m \sim \frac{\epsilon}{\alpha'} \,. \tag{4.23}$$

Let us assume that $\epsilon^2 \ll \alpha'$. Then we have $m^2 \ll M_s^2$, so that the massive adjoint chiral multiplets lie way below the string scale, and we should be able to trust the world-sheet expansion. The contribution of the low-energy field theory modes into the four-dimensional Planck scale $\widehat{M}_P^2$ is then expected to be of order m^2. This, however, is not the end of the story. The key point here is that we now have $\mathcal{N}=1$ supersymmetry, so the massive string excitations do contribute into the gauge coupling renormalization. Note, however, that each massive level is expected to give a threshold contribution of order

$$\frac{m^2}{M_s^2} \,. \tag{4.24}$$

Indeed, in the limit $m \rightarrow 0$ these threshold contributions should vanish as in this limit the $\mathcal{N}=2$ supersymmetry is restored. This, in particular, implies that the gauge theory cut-off Λ is *not* of order M_s (the lowest string excitation), but *higher*. In fact, we can estimate this cut-off as follows. We expect the ultra-violet cut-off in the gauge theory to be of order of the energy scale such that the combined contribution of the massive string thresholds lying below this scale becomes of order 1. Let n_* be the corresponding string level. Then we have:

$$n_* \, \frac{m^2}{M_s^2} \sim 1 \,, \tag{4.25}$$

[30]If we do not impose the $\mathbb{Z}_2$ symmetry under the interchange of the two gauge groups, we then have a fixed surface, where the second marginal deformation corresponds to the difference between the kinetic energies of the two $\mathrm{SU}(N)$'s [40].

that is, $n_* \sim M_s^2/m^2$. The ultra-violet cut-off is then given by:

$$\Lambda^2 \sim n_* M_s^2 \sim \frac{M_s^4}{m^2} \sim \frac{1}{\epsilon^2} \,. \tag{4.26}$$

Thus, the four-dimensional Planck scale

$$\widehat{M}_P \sim \frac{1}{\epsilon} \,. \tag{4.27}$$

Note that this is consistent with our expectation that in the conformal limit $\epsilon \to 0$ we must have $\widehat{M}_P \to \infty$.

We can understand this result intuitively in the following way. In the conformal case (that is, before blowing up the orbifold) the annulus and Möbius strip amplitudes actually give finite contributions to the 10-dimensional and 6-dimensional Planck scales depending on whether a given amplitude is untwisted or twisted. But there is no truly four-dimensional Einstein-Hilbert term. If, as in the previous subsection, we compactify the two extra dimensions untouched by the orbifold, then we have a finite four-dimensional Planck scale which goes to infinity in the decompactification limit.

Let us now discuss what happens once we blow up the orbifold (in the case where all six extra dimensions are non-compact). Note that the transverse space $\widetilde{\mathcal{M}}_6$ is smooth everywhere except for the conifold singularity (where the D3-branes are located). Attaching the external graviton lines to boundaries in, say, the annulus amplitude now *is* special as this amounts to attaching these lines at the conifold singularity. In particular, if the locations of the external insertions are away from the conifold singularity, such diagrams only renormalize the bulk operators. On the other hand, if the locations of the external insertions coincide with the conifold singularity, the corresponding diagrams renormalize the 4-dimensional Planck scale (among other terms).

Let us now understand the size of the cut-off Λ. Recall that in the orbifold background the infra-red and ultra-violet divergence structures are identical. The blow-up modifies the background, but since the blow-up size ϵ is small, at the leading order the relation between the infra-red and ultra-violet effects remains intact. Thus, let us consider the loop-channel infra-red physics. The blow-up introduces the infra-red scale m in the loop channel. This translates into an ultra-violet scale $1/\epsilon$ in the tree channel. In particular, in the tree channel the ultra-violet effects become soft around this scale. In the tree channel this scale arises due to the closed string Kaluza-Klein modes associated with the blow-up. Since the tree-channel ultra-violet effects become soft around the scale $1/\epsilon$, the brane thickness is expected to be of oder ϵ (recall that, if the brane matter is non-conformal, the brane thickness is expected to be non-vanishing so that the divergences discussed in section 3 are smoothed out).

Similarly, in the tree channel we have an *infra-red* cut-off scale m, which arises due to the fact that some of the twisted closed string states that are massless in the orbifold limit are now *massive* with masses of order m. In the loop channel this translates into the *ultra-violet* scale $\Lambda \sim 1/\epsilon$. In particular, this implies that we have a massive *open* string threshold associated with this scale (which decouples in the orbifold limit). This open string threshold is due to the fact that now the brane thickness is non-vanishing, so that we have a massive *breathing* mode with mass of order $1/\epsilon$.

Thus, as we see, the four-dimensional Planck scale in this example is controlled by the size of the blow-up. The reason why this is possible is that the blow up we are considering here is not a marginal deformation of the orbifold singularity in $\mathbb{C} \times (\mathbb{C}^2/\mathbb{Z}_2)$ but a *relevant* one. Note that above we considered an oriented type-IIB background. However, we can generalize our discussion to the orientifold of type IIB on $\mathbb{C} \times (\mathbb{C}^2/\mathbb{Z}_2)$. Note that in this case we also have an O7-plane and 8 of the corresponding D7-branes. We will come back to unoriented conifold backgrounds in section 4, where we discuss some phenomenological applications of the orientiworld models.

Before we end this subsection, let us make the following remark. To obtain the desired four-dimensional Planck scale $\widehat{M}_P \sim 10^{18}\,\mathrm{GeV}$, we must choose $\epsilon \sim \widehat{M}_P^{-1}$. Then, as we discussed in subsection 3.5, we must take $M_s \sim$ TeV — in this case the cross-over scale $r_c \sim r_H$ (the present Hubble size), while for larger M_s r_c is smaller (and smaller M_s are phenomenologically disfavored). The infra-red scale m is then of order $10^{-3}\,\mathrm{eV}$ (about an inverse millimeter). Here it is difficult to judge whether this is a worrisome feature or a bonus — the model at hand is not realistic, so we will have to postpone discussing this point until we have a more realistic looking model.

4.3 Non-conformal $\mathcal{N} = 1$ models

It is clear that in the $\mathcal{N} = 1$ orientiworld models with $\Gamma \subset \mathrm{SO}(3)$ the situation is similar to that in the $\mathcal{N} = 2$ theories discussed in the previous subsections. Let us therefore focus on the cases with $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$. As we discussed in subsection 2.4, the D3-brane gauge theories in these models are always chiral and non-conformal. Let us therefore understand the origin of the four-dimensional Planck scale in these models.

First, let us focus on the cases where all twists $g_a \in \Gamma$ have fixed point loci of real dimension 0 (that is, Γ does not have a non-trivial subgroup which is also a subgroup of SO(3), so $\Gamma = \mathbb{Z}_M$, where M is odd). Let us start with the closed string torus amplitude. The untwisted closed string states running in the loop renormalize 10-dimensional terms. However, the *twisted* closed string states give rise to a *four-dimensional* Einstein-Hilbert term (among other terms) localized at the orbifold fixed point:

$$\widehat{M}_P^2 \int d^4x \, \sqrt{-\widehat{G}}\; \widehat{R}\,. \tag{4.28}$$

The corresponding contribution to the 4-dimensional Planck scale $\widehat{M}_P^2 \sim M_s^2$. As in the previous subsection, the Klein bottle amplitude also gives a contribution of the same order of magnitude into $\widehat{M}_P^2$. This contribution is due to the untwisted closed string states running in the loop channel, while the twisted closed string states in the loop channel do not contribute ($\Omega\theta\Omega^{-1} = \theta^{-1}$).

Next, consider the one-loop open string amplitudes. The untwisted annulus and Möbius strip amplitudes renormalize the bulk terms. However, the *twisted* annulus and Möbius strip amplitudes now contribute to the renormalization of the four-dimensional Einstein-Hilbert term (as well as other terms). The corresponding contributions to the four-dimensional Planck scale $\widehat{M}_P^2$ are of order M_s^2. This is evident in the tree channel, while in the loop channel this can be understood from the fact that the massive open string excitations in $\mathcal{N} = 1$ backgrounds do contribute into the renormalization of the D3-brane gauge couplings, so that the ultra-violet cut-off Λ in the D3-brane gauge theory is of order M_s.

Here we come to an important point. The D3-brane gauge theory is non-conformal. At the orbifold point we have no ultra-violet divergences in the open string loop channel (that is, we have no infra-red divergences in the tree channel) as all twisted tadpoles are canceled. However, we do have infra-red divergences in the loop channel, which translate into ultra-violet divergences in the tree channel. In fact, as we discussed in section 3, since the D3-brane gauge theory is non-conformal, and since the D3-branes are δ-function-like, we expect ultra-violet divergences to arise in the coupling between gravity and brane matter at the tree level. On the other hand, we also expect these divergences to be smoothed out. As we will now argue, blowing up the orbifold smoothes out these divergences.[31]

Before we proceed further, let us discuss one important point concerning blowing up such orbifold singularities. For definiteness, here we will focus on a simple example, but our conclusions also apply to the general class of models we are discussing here. Thus, consider the $\mathcal{M}_6 = \mathbb{C}^3/\Gamma$ orbifold, where $\Gamma = \mathbb{Z}_3$, and the generator θ of $\mathbb{Z}_3$ acts as follows on the complex coordinates z_α, $\alpha = 1, 2, 3$, on $\mathbb{C}^3$:

$$\theta : z_\alpha \to \omega z_\alpha\,, \tag{4.29}$$

where $\omega \equiv \exp(2\pi i/3)$. We have a usual marginal blow-up in this orbifold. However, we do not have the analog of the relevant blow-up discussed in the previous subsection. Thus, we can view the above $\mathbb{C}^3/\mathbb{Z}_3$ orbifold as follows. The metric on $\mathcal{M}_6$ is given by

$$ds^2 = dr^2 + r^2\gamma_{\alpha\beta}dy^\alpha dy^\beta\,, \tag{4.30}$$

where $\gamma_{\alpha\beta}$ is a metric on a compact 5-dimensional Einstein manifold $\mathcal{Y}_5 = \mathbf{S}^5/\mathbb{Z}_3$. The generator θ of $\mathbb{Z}_3$ has the same action on the coordinates z_α on $\mathbf{S}^5$, $\sum_{\alpha=1}^3 |z_\alpha|^2 = 1$, as

[31] The fact that we should blow up the orbifold in these backgrounds was also argued in [27, 28] from a somewhat different angle.

in (4.29). Note that θ does *not* have a fixed point set on $\mathcal{Y}_5$, that is, the manifold $\mathcal{Y}_5$ is *smooth*. This is simply another way of stating the fact that the orbifold singularity in $\mathcal{M}_6$ is the same as the conifold singularity in $\mathcal{M}_6$, so blowing up the orbifold singularity is equivalent to smoothing out the conifold. And the corresponding blow-up is marginal.

The fact that the orbifold blow-up is marginal in this case implies that the blow-up does *not* introduce an infra-red cut-off in the tree channel, that is, there is no new ultra-violet threshold in the loop channel associated with the blow-up. In particular, the four-dimensional Planck scale $\widehat{M}_P$ remains of order M_s even if we consider such a marginal blow-up. Here we are assuming that the blow-up size, call it Δ, is small: $\Delta^2 \ll \alpha'$, so we can still trust the world-sheet expansion. Note that (unlike in the $\mathcal{N}=2$ cases discussed above) here the infra-red and ultra-violet divergence structures are *different*. This *a priori* allows to have an ultra-violet cut-off in the tree channel without having an infra-red cut-off in the tree channel. In particular, the blow-up provides such an ultra-violet cut-off in the tree channel — in the closed string sector we have a heavy Kaluza-Klein threshold of order $1/\Delta$ associated with the blow-up (according to our expectations in section 3, the brane thickness is then of order Δ). In the loop channel this is expected to translate into an infra-red cut-off of order

$$\mu \sim \frac{\Delta}{\alpha'} . \tag{4.31}$$

That is, after the blow-up we must have light open string states with masses of order μ. And we do. Indeed, in the gauge theory blowing up the orbifold corresponds to Higgsing along the F- and D-flat directions[32] (this is because the blow-up is marginal), that is, to an appropriate deformation of the gauge bundle. The massive open string states resulting from this Higgsing then have masses of order μ.

Note that in the above scenario we have $\widehat{M}_P \sim M_s$, so both should be of order 10^{18} GeV. Then, to have the cross-over scale not smaller than the present Hubble size, the blow-up size should be extremely small, not larger than $10^{-30}\widehat{M}_P^{-1}$. Then the scale μ is smaller than or of order 10^{-3} eV (about an inverse millimeter).

4.4 More general non-conformal $\mathcal{N}=1$ models

In the previous subsection we focused on the simple case where all twists in Γ have fixed point loci of real dimension 0. In a more general class of models we can also have twists with fixed point loci of real dimension 2. We would like to discuss this class of models next. In fact, as we will argue in the following, this class of models appears to be most suitable for the phenomenological goals we have been pursuing.

Thus, consider the orbifold group $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$ such that some twists in Γ have fixed point loci of real dimension 2. This then implies that there

[32] In the $\mathbb{Z}_3$ orientifold model, which we review in the next section, the F- and D-flat directions were discussed in detail in [48].

exists a non-trivial subgroup of Γ, call it $\overline{\Gamma}$, such that $\overline{\Gamma}\subset \mathrm{SO}(3)$ (in particular, $\overline{\Gamma}$ could be a subgroup of SU(2)). The elements of Γ that do not belong to $\overline{\Gamma}$ need not commute with the elements of $\overline{\Gamma}$. However, for our purposes here it will suffice to assume that they do (our conclusions, however, will be applicable in the general case). In this case $\Gamma = Z_M \times \overline{\Gamma}$, where M is odd, $\mathbb{Z}_M \subset \mathrm{SU}(3)$ but $\mathbb{Z}_M \not\subset \mathrm{SU}(2)$ (it then also follows that $\mathbb{Z}_M \not\subset \mathrm{SO}(3)$). Let the generator of $\mathbb{Z}_M$ be θ, while $\overline{\Gamma} = \{\overline{g}_a | a = 1,\dots,|\overline{\Gamma}|\}$. Then $\Gamma = \{\theta^k, \theta^k\overline{g}_a | k = 0,\dots,M; a = 1,\dots,|\overline{\Gamma}|\}$. Note that in this case we can view the orientifold of type IIB on $\mathbb{C}^3/\Gamma$ as the $\mathbb{Z}_M$ orbifold of the orientifold of type IIB on $\mathbb{C}^3/\overline{\Gamma}$.

Let us first discuss gravity in such a model before any blow-ups. The $\overline{g}_a$ ($a\neq 1$) twisted sectors give rise to various 6-dimensional Einstein-Hilbert terms localized on the corresponding fixed points. More precisely, if $\overline{\Gamma}\subset \mathrm{SU}(2)$, then these twisted sectors give rise to only one type of the 6-dimensional Einstein-Hilbert term, while if $\overline{\Gamma}\not\subset \mathrm{SU}(2)$, then we have more then one types of 6-dimensional Einstein-Hilbert terms (with the extra two dimensions identified with the corresponding fixed point loci). The $\theta^k\overline{g}_a$, $k\neq 0, a\neq 1$, twisted sectors can also give rise to 6-dimensional Einstein-Hilbert terms, but need not depending upon a particular orbifold group. If they do not, then they give rise to a 4-dimensional Einstein-Hilbert term. Finally, the θ^k, $k\neq 0$, twisted sectors contribute to the 4-dimensional Einstein-Hilbert term. The important point here is that we always have at least one type of a 6-dimensional Einstein-Hilbert term present in such backgrounds. Note that the 4-dimensional Planck scale $\widehat{M}_P \sim M_s$, and the 6-dimensional Planck scale $\overline{M}_P \sim M_s$. Suppose we blow up the fixed point at the origin of $\mathbb{C}^3/\Gamma$, which is associated with the $\mathbb{Z}_M$ orbifold. The brane thickness is then of order of the corresponding marginal blow-up size Δ.

Since we have 6-dimensional Einstein-Hilbert terms, before we discuss the cross-over to the 10-dimensional gravity, we must discuss a possible cross-over to the 6-dimensional gravity. However, since we do not have a tree-level infra-red cut-off in these backgrounds, following our discussion in appendix C.2, we conclude that we do not have a cross-over to the 6-dimensional gravity. The cross-over to the 10-dimensional gravity then goes as in the previous subsection. In particular, we must have an extremely small Δ in this case.

The above scenario, however, is not the only possible one. In particular, we can utilize the fact that, as we discussed in subsection 4.2, in some cases we have relevant blow-ups of the orbifold singularities associated with the twists $\overline{g}_a$. For illustrative purposes let us consider a simple example here. Thus, let $\overline{\Gamma} = \mathbb{Z}_2$, and let $\Gamma = \mathbb{Z}_3\times\mathbb{Z}_2$, where the generators θ and R of $\mathbb{Z}_3$ respectively $\mathbb{Z}_2$ have the following action on the complex coordinates z_α on $\mathbb{C}^3$:

$$\begin{aligned} &\theta z_\alpha = \omega z_\alpha\,, \qquad && \alpha = 1,2,3\,,\\ &R z_1 = z_1\,, && R z_{2,3} = -z_{2,3}\,. \end{aligned} \tag{4.32}$$

The metric on $\mathcal{M}_6 = \mathbb{C}^3/\Gamma$ is given by:

$$ds^2 = dr^2 + r^2 \gamma_{\alpha\beta} dy^\alpha dy^\beta \,, \tag{4.33}$$

where $\gamma_{\alpha\beta}$ is a metric on a compact 5-dimensional Einstein manifold $\mathcal{Y}_5 = \mathbf{S}^5/\Gamma$. The generators θ and R have the same action on the coordinates z_α on $\mathbf{S}^5$, $\sum_{\alpha=1}^3 |z_\alpha|^2 = 1$, as above. The fixed point locus of the twist R in $\mathbf{S}^5$ is a unit circle $\mathbf{S}^1$: $|z_1|^2 = 1$. The action of the twist θ on this fixed circle is given by $\theta z_1 = \omega z_1$, so the $\mathbb{Z}_3$ orbifold simply reduces the size of this circle by a factor of 3. We can therefore consider a relevant blow-up of this circle as in subsection 4.2. After this blow-up, whose size we will denote via ϵ, we no longer have a 6-dimensional Einstein-Hilbert term. Rather, both the $\mathbb{Z}_3$ and the blown-up $\mathbb{Z}_2$ orbifolds contribute into the 4-dimensional Planck scale. These contributions, however, are very different. Thus, from our previous discussions it should be clear that at the one-loop order the contributions of the $\mathbb{Z}_3$ (as well as $\mathbb{Z}_6$) twisted sectors into the 4-dimensional Planck scale $\widehat{M}_P^2$ are of order M_s^2. However the contributions from the blown-up $\mathbb{Z}_2$ orbifold are of order $1/\epsilon^2$, so assuming that $\epsilon^2 \ll \alpha'$, we have the following estimate for the 4-dimensional Planck scale:

$$\widehat{M}_P \sim \frac{1}{\epsilon} \,. \tag{4.34}$$

Let us also blow up the $\mathbb{Z}_3$ singularities (this blow-up is marginal). We will choose the corresponding blow-up size $\Delta \sim \epsilon$. It is then clear that the brane thickness is of order ϵ. Now we can take $M_s \sim$ TeV and obtain the cross-over scale (to the 10-dimensional gravity) of order of the present Hubble size. The infra-red scales

$$m \sim \frac{\epsilon}{\alpha'} \,, \tag{4.35}$$

$$\mu \sim \frac{\Delta}{\alpha'} \tag{4.36}$$

in this case are both about an inverse millimeter.

Thus, as we see, in this case the situation is similar to that discussed in the $\mathcal{N} = 2$ models in subsection 4.2 except that here the D3-brane gauge theories are non-conformal even in the orbifold limit. That is, one might hope to construct various realistic looking non-conformal chiral $\mathcal{N} = 1$ models in this context. We will discuss such models in section 4.

Before we end this subsection, the following remark is in order. Since we are discussing $\mathcal{N} = 1$ models, we expect that the D3-brane gauge theory cut-off Λ is or order M_s. It might then appear surprising that $\widehat{M}_P \gg M_s$. However, the key point here is that there are actually *two* ultra-violet cut-offs in such backgrounds. One is $\Lambda \sim M_s$, and another one is $\Lambda' \sim 1/\epsilon$. In particular, if we view the low energy modes as parts of $\mathcal{N} = 2$ multiplets corresponding to the $\mathbb{C}^3/\mathbb{Z}_2$ model, we expect that the massive string thresholds will contribute to the gauge coupling renormalization up to the scale Λ'. However, the fact that we do not have complete $\mathcal{N} = 2$ multiplets implies that it makes no sense to talk about the gauge coupling renormalization beyond the scale Λ, which is determined by the truly $\mathcal{N}=1$ supersymmetric massive string excitations that contribute to the gauge coupling renormalization starting at the lowest such excitation. In other words, the gauge theory cut-off is given by Λ as we have a tower of massive open string states starting at around M_s, while the four-dimensional gravitational cut-off is given by Λ', and gravity is weak all the way up to $\widehat{M}_P$. This is in some sense analogous to what happens in little string theories [49].

4.5 A "hybrid" scenario

Before we end this section, as an aside we would like to discuss a somewhat different scenario, where 4 of the extra dimensions are compact, but we have 2 non-compact extra dimensions. For illustrative purposes let us focus on the orbifold group $\mathbb{Z}_3 \times \mathbb{Z}_2$ discussed in the previous subsection. Thus, consider the case where the transverse space is given by $\mathcal{M}_6 = (\mathbb{C} \times \mathcal{W}_4)/\mathbb{Z}_3$, where $\mathcal{W}_4 = T^4/\mathbb{Z}_2$ is (an orbifold limit of) a K3 surface, and the complex coordinates z_1 and z_2, z_3 now parametrize $\mathbb{C}$ respectively $\mathcal{W}_4$. Since we have only two non-compact directions in $\mathcal{M}_6$, the number of D3-branes is now fixed by the untwisted tadpole cancellation conditions to be 8. This is because we have 16 O3-planes located at the corresponding fixed points on T^4. We also have an O7-plane and 8 of the corresponding D7-branes. Here we should point out that the corresponding $\mathbb{Z}_3 \times \mathbb{Z}_2$ model is not realistic as the gauge group only contains SU(2) factors (see subsection 5.5), so we do not have the $\mathrm{SU}(3)_c$ subgroup of the Standard Model. However, here we would like to use this model to discuss a point *a priori* unrelated to how realistic this model is from the Particle Physics viewpoint.

Thus, suppose the linear sizes of the K3 surface are all of order $1/M_s$. At low energies we can integrate out the K3 part and obtain a model which is equivalent to having a 3-brane in 6-dimensional bulk with infinite volume extra-dimensions. Since we have no tree-level infra-red divergences in this model (all tadpoles are canceled), following our discussion in the previous subsection, we could consider a model with no cross-over from the 4-dimensional gravity to 6-dimensional gravity. Here we should point out that the number of such "hybrid" models would be quite limited for the reasons mentioned in introduction.

5. Explicit examples

In this section we would like to discuss explicit examples of orientiworld models. As we have already mentioned in Introduction, the number of such models is infinitely large, and we have a large number of potentially interesting models to explore. Here we will not attempt to perform an exhaustive search for phenomenologically most interesting models. Rather, we will discuss some examples to get a flavor of what kind of features we can expect from the orientiworld construction (we will discuss phenomenological applications in the next section). Here we will focus on examples with the orbifold group $\Gamma \subset \mathrm{SU}(3)$ but $\Gamma \not\subset \mathrm{SO}(3)$. Such examples have been studied in [6, 9, 13]. For illustrative purposes here we will discuss some of these models.

5.1 The $\mathbb{Z}_3$ models

The simplest choice of the orbifold group Γ in the present context is $\Gamma = \mathbb{Z}_3 = \{1, \theta, \theta^2\}$, where the generator θ of $\mathbb{Z}_3$ acts on the complex coordinates z_α, $\alpha = 1, 2, 3$, on $\mathbb{C}^3$ as follows:

$$\theta z_\alpha = \omega z_\alpha \,, \tag{5.1}$$

where $\omega \equiv \exp(2\pi i/3)$. At the origin of $\mathbb{C}^3/\Gamma$ we can place an O3$^-$- or O3$^+$-plane. Let $\eta = -1$ in the former case, while $\eta = +1$ in the latter case. The twisted tadpole cancellation requires that [6]

$$\mathrm{Tr}(\gamma_\theta) = 4\eta \,. \tag{5.2}$$

We can then place $(3N + 4\eta)$ D3-branes on top of the O3-plane, and choose

$$\gamma_\theta = \mathrm{diag}(\omega I_N, \omega^{-1} I_N, I_{N+4\eta}) \,. \tag{5.3}$$

The massless spectrum of the resulting model is given in table 1. Note that the U(1) factor is anomalous and is actually broken at the tree level as we discussed in

Model	Gauge Group	Charged Chiral Multiplets	Twisted Sector Chiral Multiplets
$\mathbb{Z}_3$	$\mathrm{U}(N) \otimes G_\eta(N+4\eta)$	$3 \times (\mathbb{R}_\eta, \mathbf{1})(+2)$ $3 \times (\overline{\mathbf{N}}, \mathbf{N+4\eta})(-1)$	1
$\mathbb{Z}_7$	$\mathrm{U}(N) \otimes \mathrm{U}(N) \otimes \mathrm{U}(N) \otimes$ $G_\eta(N-4\eta)$	$(\mathbb{R}_\eta, \mathbf{1}, \mathbf{1}, \mathbf{1})(+2, 0, 0)$ $(\mathbf{N}, \mathbf{1}, \mathbf{1}, \mathbf{N-4\eta})(+1, 0, 0)$ $(\overline{\mathbf{N}}, \mathbf{N}, \mathbf{1}, \mathbf{1})(-1, +1, 0)$ $(\overline{\mathbf{N}}, \overline{\mathbf{N}}, \mathbf{1}, \mathbf{1})(-1, -1, 0)$ plus cyclic permutations of the $\mathrm{U}(N) \otimes \mathrm{U}(N) \otimes \mathrm{U}(N)$ representations	3
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$\mathrm{U}(N) \otimes \mathrm{U}(N) \otimes \mathrm{U}(N) \otimes$ $\mathrm{U}(N+2\eta) \otimes G_\eta(N)$	$(\mathbb{R}_\eta, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(+2, 0, 0, 0)$ $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{N})(-1, 0, 0, 0)$ $(\mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}, \mathbf{N+2\eta}, \mathbf{1})(0, -1, 0, +1)$ $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}}, \overline{\mathbf{N+2\eta}}, \mathbf{1})(0, 0, -1, -1)$ $(\mathbf{1}, \mathbf{N}, \mathbf{N}, \mathbf{1}, \mathbf{1})(0, +1, +1, 0)$ plus cyclic permutations of the $\mathrm{U}(N) \otimes \mathrm{U}(N) \otimes \mathrm{U}(N)$ representations	7

Table 1: The massless spectra of $\mathcal{N} = 1$ orientifolds of type IIB on $\mathbb{C}^3/\mathbb{Z}_3$, $\mathbb{C}^3/\mathbb{Z}_7$ and $\mathbb{C}^3/(\mathbb{Z}_3 \times \mathbb{Z}_3)$. Here $\mathbb{R}_\eta = \mathbf{A}$ (two-index $N(N-1)/2$ dimensional anti-symmetric representation of U(N)) for $\eta = -1$, and $\mathbb{R}_\eta = \mathbf{S}$ (two-index $N(N+1)/2$ dimensional symmetric representation of U(N)) for $\eta = +1$. The U(1) charges of the states in the 33 open string sector are given in parentheses. The untwisted closed string sector states are not shown.

subsection 2.4. Let $\Phi_\alpha = 3 \times (\mathbb{R}_\eta, \mathbf{1})$, and $P_\alpha = 3 \times (\overline{\mathbf{N}}, \mathbf{N} + \mathbf{4}\eta)$. Then the tree-level superpotential in this model is given by:

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma} \Phi_\alpha P_\beta P_\gamma + \cdots , \tag{5.4}$$

where the ellipses stand for non-renormalizable couplings.

An interesting case to consider is $\eta = -1$ and $N = 5$ (in this case we have 11 D3-branes placed on top of the O3$^-$-plane). The gauge group of this model is SU(5), and we have 3 chiral generations transforming in $\mathbf{10}$ plus $\overline{\mathbf{5}}$ of SU(5). A compact version of this model was originally constructed in [46]. Even though we have three generations and an appealing gauge group in this model, phenomenologically it is still not very interesting as we have no way of breaking the "grand unified" SU(5) gauge group down to the Standard Model gauge group $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ — we have no Higgs field in an appropriate higher dimensional representation (such as the adjoint Higgs).

5.2 The $\mathbb{Z}_7$ models

Another simple choice of the orbifold group Γ is $\Gamma = \mathbb{Z}_7$, where the generator g of $\mathbb{Z}_7$ acts on the complex coordinates z_α, $\alpha = 1, 2, 3$, on $\mathbb{C}^3$ as follows:

$$g z_1 = \alpha z_1 , \qquad g z_2 = \alpha^2 z_2 , \qquad g z_3 = \alpha^4 z_3 , \tag{5.5}$$

where $\alpha \equiv \exp(2\pi i/7)$. At the origin of $\mathbb{C}^3/\Gamma$ we can place an O3$^-$- or O3$^+$-plane. As before, let $\eta = -1$ in the former case, while $\eta = +1$ in the latter case. The twisted tadpole cancellation requires that [6]

$$\mathrm{Tr}(\gamma_g) = -4\eta . \tag{5.6}$$

We can then place $(7N - 4\eta)$ D3-branes on top of the O3-plane, and choose

$$\gamma_g = \mathrm{diag}(\alpha I_N, \alpha^{-1} I_N, \alpha^2 I_N, \alpha^{-2} I_N, \alpha^4 I_N, \alpha^{-4} I_N, I_{N-4\eta}) . \tag{5.7}$$

The massless spectrum of the resulting model is given in table 1. Note that the U(1) factors are anomalous and are actually broken at the tree level as we discussed in subsection 2.4. Let $\Phi_1 = (\mathbf{1}, \mathbf{1}, \mathbb{R}_\eta, \mathbf{1})$, $P_1 = (\mathbf{N}, \mathbf{1}, \mathbf{1}, \mathbf{N} + \mathbf{4}\eta)$, $R_1 = (\overline{\mathbf{N}}, \mathbf{N}, \mathbf{1}, \mathbf{1})$, $Q_1 = (\mathbf{1}, \overline{\mathbf{N}}, \overline{\mathbf{N}}, \mathbf{1})$, plus cyclic permutations of the $SU(N) \otimes SU(N) \otimes SU(N)$ representations which define Φ_α, P_α, R_α, Q_α with $\alpha = 2, 3$. The tree-level superpotential in this model is given by:

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma} P_\alpha P_\beta Q_\gamma + \epsilon_{\alpha\beta\gamma} Q_\alpha R_\beta \Phi_\gamma + \epsilon_{\alpha\beta\gamma} R_\alpha R_\beta R_\gamma + \cdots , \tag{5.8}$$

where the ellipses stand for non-renormalizable couplings.

Note that in this model we have a cyclic $\mathbb{Z}_3$ symmetry corresponding to the permutations of the $SU(N) \otimes SU(N) \otimes SU(N)$ quantum numbers. In fact, this is a symmetry of the background itself. In the next subsection we will consider further orbifolding the $\mathbb{Z}_7$ models by this symmetry.

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5.3 A non-abelian orbifold

Thus, consider the orbifold group $\mathbb{Z}_3 = \{1, \theta, \theta^2\}$ whose generator θ has the following action on the complex coordinates z_α:

$$\theta z_1 = z_2\,, \qquad \theta z_2 = z_3\,, \qquad \theta z_3 = z_1\,. \tag{5.9}$$

That is, θ permutes the complex coordinates z_α. Note that θ does not commute with the generator g of the $\mathbb{Z}_7$ orbifold group discussed in the previous subsection:

$$\begin{aligned} &\theta g \theta^{-1} = g^2\,, \qquad \theta g^2 \theta^{-1} = g^4\,, \qquad \theta g^4 \theta^{-1} = g\,, \\ &\theta g^3 \theta^{-1} = g^6\,, \qquad \theta g^6 \theta^{-1} = g^5\,, \qquad \theta g^5 \theta^{-1} = g^3\,. \end{aligned} \tag{5.10}$$

Also note that

$$(\theta g^k)^3 = (\theta^2 g^k)^3 = 1\,, \qquad k = 0, \dots, 6\,. \tag{5.11}$$

Thus, θ and g together generate a non-abelian group Γ_* with $|\Gamma_*| = 21$. This orbifold group is a subgroup of SU(3).

Let us consider the orientiworld model corresponding to the $\mathbb{C}^3/\Gamma_*$ orbifold. The eigenvalues of θg^k and $\theta^2 g^k$, $k = 0, \dots, 6$, are $1, \omega, \omega^{-1}$. The twisted tadpole cancellation conditions then imply that [6]

$$\mathrm{Tr}(\gamma_\theta g^k) = \mathrm{Tr}(\gamma_\theta^2 g^k) = -2\eta\,, \qquad k = 0, \dots, 6\,. \tag{5.12}$$

We can therefore choose:

$$\begin{aligned} \gamma_g &= \mathrm{diag}(\alpha I_N, \alpha^{-1} I_N, \alpha^2 I_N, \alpha^{-2} I_N, \alpha^4 I_N, \alpha^{-4} I_N, I_{N-4\eta})\,, \\ \gamma_\theta &= \mathrm{diag}(I_{2N} \times \mathcal{P}_3, \omega I_n, \omega^{-1} I_n, I_{n-2\eta})\,, \end{aligned} \tag{5.13}$$

where $N = 3n + 2\eta$, and $\mathcal{P}_3$ is a 3×3 permutation matrix.

The resulting $\mathcal{N} = 1$ supersymmetric theory has the gauge group $\mathrm{SU}(N) \otimes \mathrm{SU}(n) \otimes G_\eta(n - 2\eta)$ (see table 1 for notations), where we have dropped the U(1) factors. The charged matter is given by

$$\begin{aligned} \Phi &= (\mathbb{R}_\eta, \mathbf{1}, \mathbf{1})\,, \\ \Sigma &= (\mathbf{N}, \mathbf{n}, \mathbf{1})\,, \\ \widetilde{\Sigma} &= (\mathbf{N}, \overline{\mathbf{n}}, \mathbf{1})\,, \\ P &= (\mathbf{N}, \mathbf{1}, \mathbf{n} - \mathbf{2}\eta)\,, \\ R &= (\mathbf{Adj}, \mathbf{1}, \mathbf{1})\,, \\ Q_\eta &= (\mathbb{R}_\eta, \mathbf{1}, \mathbf{1})\,, \\ Q_{-\eta} &= (\mathbb{R}_{-\eta}, \mathbf{1}, \mathbf{1})\,, \end{aligned} \tag{5.14}$$

where **Adj** stands for the adjoint representation of SU(N). The tree-level superpotential is given by:

$$\mathcal{W} = \Phi Q_\eta R + P P Q_{-\eta} + \Sigma\widetilde{\Sigma} Q_\eta + \Sigma\widetilde{\Sigma} Q_{-\eta} + \cdots\,, \tag{5.15}$$

where the ellipses stand for non-renormalizable couplings.

An interesting feature of this model is that we have matter in the *adjoint* of the SU(N) subgroup. Such higher dimensional representations are necessary in grand unified models to break the grand unified gauge group down to that of the Standard Model. Note, however, that in this model the appearance of the adjoint is achieved at the cost of having only one "generation" instead of three.[33]

5.4 The $\mathbb{Z}_3 \times \mathbb{Z}_3$ models

Next, let us consider the case with $\Gamma = \mathbb{Z}_3 \times \mathbb{Z}_3$. Let g_1 and g_2 be the generators of the two $\mathbb{Z}_3$ subgroups. Their action on the complex coordinates z_α is given by:

$$\begin{aligned} g_1 z_1 &= \omega z_1 \,, \qquad & g_1 z_2 &= \omega^{-1} z_2 \,, \qquad & g_1 z_3 &= z_3 \,, \\ g_2 z_1 &= z_1 \,, \qquad & g_2 z_2 &= \omega z_2 \,, \qquad & g_2 z_3 &= \omega^{-1} z_3 \,. \end{aligned} \tag{5.16}$$

The twisted tadpole cancellation conditions read [9]:

$$\begin{aligned} \mathrm{Tr}(\gamma_{g_1}) = \mathrm{Tr}(\gamma_{g_2}) &= \mathrm{Tr}(\gamma_{g_1 g_2}) = -2\eta \,, \\ \mathrm{Tr}(\gamma_{g_1 g_2^2}) &= \mathrm{Tr}(\gamma_{g_1^2 g_2}) = 4\eta \,. \end{aligned} \tag{5.17}$$

We can place $(9N + 4\eta)$ D3-branes on top of the O3-plane located at the origin of $\mathbb{C}^3/\Gamma$, and choose

$$\begin{aligned} \gamma_{g_1} &= \mathrm{diag}(\omega I_{3N+2\eta}, \omega^{-1} I_{3N+2\eta}, I_{3N}) \,, \\ \gamma_{g_2} &= \mathrm{diag}(\omega I_{N+2\eta}, \omega^{-1} I_N, I_N, \omega^{-1} I_{N+2\eta}, \omega I_N, I_N, \omega I_N, \omega^{-1} I_N, I_N) \,. \end{aligned} \tag{5.18}$$

The massless spectrum of the resulting model is given in table 1 (the U(1) factors should be dropped). Let $\chi_1 = (\mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}, \mathbf{1}, \mathbf{N})$, $P_1 = (\mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}, \mathbf{N} + \mathbf{2}\eta, \mathbf{1})$, $Q_1 = (\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}}, \overline{\mathbf{N} + \mathbf{2}\eta}, \mathbf{1})$, $R_1 = (\mathbf{1}, \mathbf{N}, \mathbf{N}, \mathbf{1}, \mathbf{1})$, plus cyclic permutations of the SU(N) $\otimes$ SU(N) $\otimes$ SU(N) representations which define $\chi_\alpha, P_\alpha, R_\alpha, Q_\alpha$ with $\alpha = 2, 3$. The tree-level superpotential in this model is given by:

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma} \chi_\alpha \chi_\beta R_\gamma + \epsilon_{\alpha\beta\gamma} P_\alpha Q_\beta R_\gamma + \cdots \,, \tag{5.19}$$

where the ellipses stand for non-renormalizable couplings.

Note that in this model we also have a cyclic $\mathbb{Z}_3$ symmetry whose generator θ acts on the complex coordinates z_α as in (5.9). We can therefore consider further orbifolding by this symmetry. Note that θ does not commute with the original $\mathbb{Z}_3 \times \mathbb{Z}_3$ generators g_1, g_2. In fact, g_1 and θ generate a non-abelian discrete group[34]

[33] Compact models with adjoint representations were recently discussed in [47] in the context of type-IIA orientifolds. In particular, an SU(5) model with 2 chiral generations was constructed in[47], but, to the best of our knowledge, a grand unified orientifold model with 3 chiral generations has not been constructed yet.

[34] The group $\Delta(3 \cdot 3^2)$ is a semi-direct product of $\mathbb{Z}_3$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$. It is the $n = 3$ member of the infinite series referred to as $\Delta(3 \cdot n^2)$, where $\Delta(3 \cdot n^2)$ is a semi-direct product of $\mathbb{Z}_3$ and $\mathbb{Z}_n \times \mathbb{Z}_n$.

$\Gamma = \Delta(3 \cdot 3^2)$ with $|\Gamma| = 27$, which is a subgroup of SU(3). The resulting model has the gauge group $\mathrm{SU}(3N-2\eta) \times \mathrm{SU}(N)^4 \times G_\eta(N-2\eta)$, and the matter content can be found in [7]. In particular, we have matter in $\mathbb{R}_\eta$ and $\mathbb{R}_{-\eta}$ of $\mathrm{SU}(3N-2\eta)$ as well as bifundamentals (but no adjoints).

5.5 The $\mathbb{Z}_3 \times \mathbb{Z}_2$ models

So far we have discussed cases without D7-branes. Let us now consider cases with D7-branes. Thus, consider the case with $\Gamma = \mathbb{Z}_3 \times \mathbb{Z}_2$. Here the action of the generators θ of $\mathbb{Z}_3 \subset \Gamma$ and R of $\mathbb{Z}_2 \subset \Gamma$ on the complex coordinates z_α is given by:

$$\begin{aligned} &\theta z_\alpha = \omega z_\alpha \,, \\ &R z_1 = -z_1 \,, \qquad R z_2 = -z_2 \,, \qquad R z_3 = z_3 \,. \end{aligned} \tag{5.20}$$

Note that now we have an O3-plane as well as a single O7-plane (the O7-plane is located at the origin of the z_3 complex plane). The untwisted tadpole cancellation conditions require the presence of 8 of the corresponding D7-branes. The twisted tadpole cancellation conditions read [9] ($k = 1, 2$):

$$\begin{aligned} \mathrm{Tr}(\gamma_{R,3}) &= \mathrm{Tr}(\gamma_{R,7}) = 0 \,, \\ \mathrm{Tr}(\gamma_{\theta^k R,3}) &= \mathrm{Tr}(\gamma_{\theta^k R,7}) = 0 \,, \\ \mathrm{Tr}(\gamma_{\theta^k,3}) &= \mathrm{Tr}(\gamma_{\theta^k,7}) = -4 \,. \end{aligned} \tag{5.21}$$

We can place $(6N-4)$ D3-branes on top of the O3-plane, and choose

$$\begin{aligned} \gamma_{\theta,3} &= \mathrm{diag}(\omega \mathbf{I}_{2N}, \omega^2 \mathbf{I}_{2N}, \mathbf{I}_{2N-4}) \,, \\ \gamma_{R,3} &= \mathrm{diag}(i, -i) \times \mathbf{I}_{3N-2} \,, \\ \gamma_{\theta,7} &= \mathrm{diag}(\omega \mathbf{I}_4, \omega^2 \mathbf{I}_4) \,, \\ \gamma_{R,7} &= \mathrm{diag}(i, -i) \times \mathbf{I}_4 \,. \end{aligned} \tag{5.22}$$

The spectrum of this model is given in table 2 (the 33 U(1) factors should be dropped). Let $\Phi_a = 2 \times (\mathbf{A}, \mathbf{1}, \mathbf{1})_{33}$, $\widetilde{\Phi}_a = 2 \times (\mathbf{1}, \overline{\mathbf{A}}, \mathbf{1})_{33}$, $Q_a = 2 \times (\overline{\mathbf{N}}, \mathbf{1}, \overline{\mathbf{N}-\mathbf{2}})_{33}$, $\widetilde{Q}_a = 2 \times (\mathbf{1}, \mathbf{N}, \mathbf{N}-\mathbf{2})_{33}$. Next, let $P = (\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1})_{33}$, $R = (\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N}-\mathbf{2})_{33}$, $\widetilde{R} = (\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}-\mathbf{2}})_{33}$, $S = (\mathbf{N}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{1})_{37}$, $\widetilde{S} = (\mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{37}$. We will also use $T = (\mathbf{1}, \mathbf{1}, \mathbf{N}-\mathbf{2}; \mathbf{1}, \mathbf{2})_{37}$, $\widetilde{T} = (\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}-\mathbf{2}}; \mathbf{2}, \mathbf{1})_{37}$, $U = (\mathbf{2}, \mathbf{2})_{77}$ (see table 2). Here $a = 1, 2$ labels the multiplicity of states in the 33 open string sector. The tree-level superpotential in this model is given by:

$$\begin{aligned} \mathcal{W} = \; & \Phi_1 Q_2 R + \Phi_2 Q_1 R + \widetilde{\Phi}_1 \widetilde{Q}_2 \widetilde{R} + \widetilde{\Phi}_2 \widetilde{Q}_1 \widetilde{R} + \\ & + Q_1 \widetilde{Q}_2 P + Q_2 \widetilde{Q}_1 P + T \widetilde{T} U + T \widetilde{S} \widetilde{R} + \widetilde{T} S R + \cdots \,, \end{aligned} \tag{5.23}$$

where the ellipses stand for non-renormalizable couplings.

Model	Gauge Group	Charged Chiral Multiplets	Twisted Sector Chiral Multiplets
$\mathbb{Z}_3 \times \mathbb{Z}_2$	$[\mathrm{U}(N) \otimes \mathrm{U}(N) \otimes \mathrm{U}(N-2)]_{33}$ $\otimes[\mathrm{U}(2) \otimes \mathrm{U}(2)]_{77}$	$2 \times (\mathbf{A}, \mathbf{1}, \mathbf{1})(+2,0,0)_{33}$ $2 \times (\mathbf{1}, \overline{\mathbf{A}}, \mathbf{1})(0,-2,0)_{33}$ $2 \times (\overline{\mathbf{N}}, \mathbf{1}, \overline{\mathbf{N-2}})(-1,0,-1)_{33}$ $2 \times (\mathbf{1}, \mathbf{N}, \mathbf{N-2})(0,+1,+1)_{33}$ $(\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1})(+1,-1,0)_{33}$ $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N-2})(-1,0,+1)_{33}$ $(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N-2}})(0+1,-1)_{33}$ $2 \times (\mathbf{1}, \mathbf{1})(+2,0)_{77}$ $2 \times (\mathbf{1}, \mathbf{1})(0,-2)_{77}$ $(\mathbf{2}, \mathbf{2})(+1,-1)_{77}$ $(\mathbf{N}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{1})(+1,0,0;+1,0)_{37}$ $(\mathbf{1}, \mathbf{1}, \mathbf{N-2}; \mathbf{1}, \mathbf{2})(0,0,+1;0,+1)_{37}$ $(\mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}; \mathbf{1}, \mathbf{2})(0,-1,0;0,-1)_{37}$ $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N-2}}; \mathbf{2}, \mathbf{1})(0,0,-1;-1,0)_{37}$	3
$\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	$[\mathrm{U}(N) \otimes \mathrm{Sp}(N-2)]_{33} \otimes$ $\bigotimes_{\alpha=1}^{3} \mathrm{U}(2)_{7_\alpha 7_\alpha}$	$3 \times (\mathbf{A}, \mathbf{1})(+2)_{33}$ $3 \times (\overline{\mathbf{N}}, \mathbf{N-2})(-1)_{33}$ $3 \times (\mathbf{1}_\alpha)(+2_\alpha)_{7_\alpha 7_\alpha}$ $(\mathbf{N}, \mathbf{1}; \mathbf{2}_\alpha)(+1;+1_\alpha)_{37_\alpha}$ $(\mathbf{1}, \mathbf{N-2}; \mathbf{2}_\alpha)(0;-1_\alpha)_{37_\alpha}$ $(\mathbf{2}_\alpha; \mathbf{2}_\beta)(+1_\alpha;+1_\beta)_{7_\alpha 7_\beta}$	7

Table 2: The massless spectra of the $\mathcal{N}=1$ orientifolds of type IIB on $\mathbb{C}^3/(\mathbb{Z}_3 \times \mathbb{Z}_2)$ and $\mathbb{C}^3/(\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$. The semi-colon in the column "Charged Chiral Multiplets" separates the 33 and $7_\alpha 7_\alpha$ representations. We are also using some of the notations from table 1. The untwisted closed string sector states are not shown.

5.6 The $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ models

Let us consider one more case with D7-branes. In particular, here we would like to discuss a model with three sets of D7-branes. Thus, consider the case with $\Gamma = \mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, where the generators θ of $\mathbb{Z}_3$, R_1 of the first $\mathbb{Z}_2$ subgroup and R_2 of the second $\mathbb{Z}_2$ subgroup act as follows:

$$\begin{aligned} \theta z_\alpha &= \omega z_\alpha \,, \\ R_\alpha z_\beta &= -(-1)^{\delta_{\alpha\beta}} z_\beta \,, \end{aligned} \tag{5.24}$$

where $R_3 \equiv R_1 R_2$. Note that now we have an O3-plane as well as three different $\mathrm{O7}_\alpha$-planes (the $\mathrm{O7}_\alpha$-plane is located at the origin of the z_α complex plane). The untwisted tadpole cancellation conditions require the presence of three sets of the corresponding $\mathrm{D7}_\alpha$-branes with 8 D7-branes in each set. The twisted tadpole cancellation conditions read [9] ($k = 1, 2$):

$$\mathrm{Tr}(\gamma_{R_\alpha,3}) = \mathrm{Tr}(\gamma_{R_\alpha,7_\beta}) = 0 \,,$$

$$\begin{aligned}\mathrm{Tr}(\gamma_{\theta^k R_\alpha,3}) &= \mathrm{Tr}(\gamma_{\theta^k R_\alpha,7_\beta}) = 0\,,\\ \mathrm{Tr}(\gamma_{\theta^k,3}) &= \mathrm{Tr}(\gamma_{\theta^k,7_\alpha}) = -4\,.\end{aligned}\tag{5.25}$$

We can place $(6N-4)$ D3-branes on top of the O3-plane, and choose

$$\begin{aligned}\gamma_{\theta,3} &= \mathrm{diag}(\omega\mathbf{I}_{2N}, \omega^2\mathbf{I}_{2N}, \mathbf{I}_{2N-4})\,,\\ \gamma_{R_\alpha,3} &= i\sigma_\alpha \times \mathbf{I}_{3N-2}\,,\end{aligned}\tag{5.26}$$

where σ_α are the 2×2 Pauli matrices. The action on the $\mathrm{D7}_\alpha$ Chan-Paton charges is similar except that their number is fixed. The spectrum of this model is given in table 2. The 33 U(1) factor is anomalous, and is broken at the tree level. Let $\Phi_\alpha = 3\times(\mathbf{A},\mathbf{1})_{33}$, $\chi_\alpha = 3\times(\overline{\mathbf{N}},\mathbf{N-2})_{33}$, $P^\alpha = (\mathbf{N},\mathbf{1};\mathbf{2}_\alpha)_{37_\alpha}$, $R^\alpha = (\mathbf{1},\mathbf{N-2};\mathbf{2}_\alpha)_{37_\alpha}$ (see table 2). The tree-level superpotential in this model is given by:

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma}\Phi_\alpha\chi_\beta\chi_\gamma + \chi_\alpha P^\alpha R^\alpha + \cdots\,,\tag{5.27}$$

where the ellipses stand for non-renormalizable couplings.

6. A three-family SU(6) model

In the $\mathbb{Z}_3\times\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold models we discussed in the previous section it is particularly interesting to discuss the case $N=6$ (see subsection 5.6). The corresponding $\mathcal{N}=1$ supersymmetric model has the gauge group $[\mathrm{SU}(6)\otimes\mathrm{Sp}(4)]_{33}\otimes\bigotimes_{\alpha=1}^{3}\mathrm{SU}(2)_{7_\alpha 7_\alpha}$. As we pointed out at the end of section 2, at the quantum level the 77 gauge symmetry becomes part of the four-dimensional gauge symmetry. Note that we have deleted the 77 U(1) factors, which are anomalous and are broken via the generalized Green-Schwarz mechanism.

The charged chiral matter multiplets in this model are given by:

$$\begin{aligned}\Phi_\alpha &= 3\times(\mathbf{15},\mathbf{1})_{33}\,,\\ \chi_\alpha &= 3\times(\overline{\mathbf{6}},\mathbf{4})_{33}\,,\\ P^\alpha &= (\mathbf{6},\mathbf{1};\mathbf{2}_\alpha)_{37_\alpha}\,,\\ R^\alpha &= (\mathbf{1},\mathbf{4};\mathbf{2}_\alpha)_{37_\alpha}\,,\\ T^{\alpha\beta} &= (\mathbf{2}_\alpha;\mathbf{2}_\beta)_{7_\alpha 7_\beta}\,.\end{aligned}\tag{6.1}$$

The tree-level superpotential is given by

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma}\Phi_\alpha\chi_\beta\chi_\gamma + \chi_\alpha P^\alpha R^\alpha + \cdots\,,\tag{6.2}$$

where the ellipses stand for non-renormalizable couplings.

We can Higgs the 77 gauge group down to the SU(2) diagonal subgroup using the $T^{\alpha\beta}$ matter multiplets. The resulting model now has the gauge group $\mathrm{SU}(6) \otimes \mathrm{Sp}(4) \otimes \mathrm{SU}(2)$ and the charged matter multiplets

$$\begin{aligned}
\Phi_\alpha &= 3 \times (\mathbf{15}, \mathbf{1}, \mathbf{1})\,, \\
\chi_\alpha &= 3 \times (\overline{\mathbf{6}}, \mathbf{4}, \mathbf{1})\,, \\
P^\alpha &= 3 \times (\mathbf{6}, \mathbf{1}, \mathbf{2})\,, \\
R^\alpha &= 3 \times (\mathbf{1}, \mathbf{4}, \mathbf{2})\,, \\
T &= (\mathbf{1}, \mathbf{1}, \mathbf{3})\,,
\end{aligned} \tag{6.3}$$

and the tree-level superpotential is still given by (6.2).

Note that we cannot break the SU(6) gauge group down to that of the Standard Model as we have no appropriate higher dimensional Higgs fields such as an adjoint Higgs. However, following [10], we can still break the $\mathrm{SU}(6) \otimes \mathrm{Sp}(4) \otimes \mathrm{SU}(2)$ gauge group down to $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_w \otimes \mathrm{U}(1)_Y$. This can be done as follows. Note that under the breaking $\mathrm{SU}(6) \otimes \mathrm{Sp}(4) \supset [\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)] \times [\mathrm{SU}(2) \otimes \mathrm{SU}(2)]$ we have the following branchings (the U(1) charges are given in parenthesis):

$$\begin{aligned}
(\mathbf{15}, \mathbf{1}) &= (\mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1})(+2) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(-4) \oplus (\mathbf{4}, \mathbf{2}, \mathbf{1}, \mathbf{1})(-1)\,, \\
(\overline{\mathbf{6}}, \mathbf{4}) &= (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1})(-1) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{2})(-1) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})(+2) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})(+2)
\end{aligned} \tag{6.4}$$

Now consider giving non-zero vacuum expectation values to $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(-4)$ in Φ_1 and $(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})(+2)$ in χ_1 (it is not difficult to see that we have the required F- and D-flat directions). This Higgsing breaks the $\mathrm{SU}(6) \otimes \mathrm{Sp}(4) \otimes \mathrm{SU}(2)$ gauge symmetry down to $\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$. The charged matter is given by:[35]

$$\begin{aligned}
\Phi_\alpha &= 3 \times (\mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1})\,, \\
\widetilde{\Phi}_1 &= (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{1})\,, \\
\chi_\alpha &= 3 \times (\overline{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{1})\,, \\
\widetilde{\chi}_1 &= (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1})\,, \\
\widehat{\chi}_1 &= (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})\,, \\
P^\alpha &= 3 \times (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2})\,, \\
\widehat{P}^1 &= (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})\,, \\
R^\alpha &= 3 \times (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})\,, \\
\widehat{R}^1 &= (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})\,, \\
T &= (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})\,,
\end{aligned} \tag{6.5}$$

and the superpotential is given by:

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma} \Phi_\alpha \chi_\beta \chi_\gamma + \chi_1 P^1 \widehat{R}^1 + \widetilde{\chi}_1 P^1 R^1 + \widehat{\chi}_1 \widehat{P}^1 R^1 + \cdots\,. \tag{6.6}$$

[35] Determining the matter content after Higgsing requires some care as we have non-trivial couplings in the superpotential.

Note that this superpotential does not contain R^2 or R^3. Let us give a non-vanishing vacuum expectation value to R^2. This breaks the gauge group down to the *Pati-Salam* gauge group $\mathrm{SU}(4)_c \otimes \mathrm{SU}(2)_w \otimes \mathrm{SU}(2)_R$. The charged matter is given by (we have interchanged $\mathbf{4}$ and $\overline{\mathbf{4}}$ of SU(4)):

$$
\begin{aligned}
\Phi_\alpha &= 3 \times (\mathbf{6}, \mathbf{1}, \mathbf{1})\,, \\
\widetilde{\Phi}_1 &= (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})\,, \\
\chi_\alpha &= 3 \times (\mathbf{4}, \mathbf{2}, \mathbf{1})\,, \\
\widetilde{\chi}_1 &= (\mathbf{4}, \mathbf{1}, \mathbf{2})\,, \\
\widehat{\chi}_1 &= (\mathbf{1}, \mathbf{2}, \mathbf{2})\,, \\
P^\alpha &= 3 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})\,, \\
\widehat{P}^1 &= (\mathbf{1}, \mathbf{2}, \mathbf{2})\,, \\
R^1, R^3 &= 2 \times (\mathbf{1}, \mathbf{1}, \mathbf{3})\,, \\
\widehat{R}^1 &= (\mathbf{1}, \mathbf{2}, \mathbf{2})\,, \\
T &= (\mathbf{1}, \mathbf{1}, \mathbf{3})\,,
\end{aligned}
\tag{6.7}
$$

and the superpotential is given by (6.6).

We can identify χ_α and P^α as three generations of quarks and leptons in the Pati-Salam model. The fields $\widetilde{\chi}_1$ and $\widetilde{\Phi}_1$ are the Pati-Salam Higgs fields, which serve the purpose of breaking the Pati-Salam gauge group down to $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_w \otimes \mathrm{U}(1)_Y$ (that is, non-vanishing vacuum expectation values of $\widetilde{\chi}_1$ and $\widetilde{\Phi}_1$ break $\mathrm{SU}(4)_c \otimes \mathrm{SU}(2)_R$ down to $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y$). The field $\widehat{R}^1$ gives rise to the electroweak Higgs doublets. In fact, as can be seen from the superpotential (6.6), the third generation (that is, the top quark generation) is identified with χ_1 and P^1. The extra triplets R^1, R^3, T should acquire soft masses upon supersymmetry breaking (these masses are presumably of order $M_s \sim$ TeV). Here we note that, upon integrating out these states, it might be possible to generate higher dimensional operators responsible for giving masses to the other two chiral generations. Potentially dangerous states are the anti-symmetric states Φ_α. They too could acquire masses upon supersymmetry breaking. However, it needs to be checked whether there are no dangerous baryon number violating couplings due to these states. This is, however, outside of the scope of this paper.

Thus, as we see, in the orientiworld context we have constructed a truly 3-family Pati-Salam model with the correct Higgs content. In fact, we even have the correct couplings in the superpotential for one generation to be much heavier then the other two. To the best of our knowledge, this model is the closest to the Minimal Supersymmetric Standard Model constructed via the orientifold construction.

Before we end this section, let us make the following remark. Here we discussed this model at the orbifold point. To obtain four-dimensional gravity, as we discussed in section 4, we need to consider relevant blow-ups of the $\mathbb{Z}_2$ twists, that is, we need to consider the corresponding conifold background. In the process of blowing up these twists, some of the massless states acquire non-zero masses. However, these masses are about 10^{-3} eV, so the above discussion is not expected to be modified substantially. It would be interesting to understand whether this scale 10^{-3} eV has anything to do with the neutrino masses (note that in the Pati-Salam model we have right-handed neutrinos).

7. Discussion

In the orientiworld framework we discussed in this paper the Standard Model gauge and matter fields are localized on (a collection of) D3-branes embedded in infinite-volume extra space. This transverse space can be an orbifold or a more general conifold. The four-dimensional gravity on D3-branes is reproduced due to a four-dimensional Einstein-Hilbert term induced at the quantum level. In particular, gravity can be four-dimensional up to the cross-over distance scale r_c, which can be as large as the present Hubble size. Not surprisingly, this requires a hierarchy of scales. Here we would like to comment on this hierarchy.

For definiteness let us focus on the scenario we discussed in subsection 4.4. In this scenario, where the transverse space is a conifold obtained by relevant blow-ups of certain orbifold singularities, the string scale M_s is of order TeV, while the four-dimensional Planck scale $\widehat{M}_P \sim 1/\epsilon$, where ϵ is the size of these blow-ups. The aforementioned hierarchy then is in having two very different distance scales, namely, the string length $1/M_s$ and the blow-up size ϵ. One possibility is that the size of the blow-up is determined dynamically upon supersymmetry breaking. So we must understand supersymmetry breaking in the orientiworld framework.

One possibility for obtaining non-supersymmetric orientiworld models is to start from $\mathcal{N} = 1$ supersymmetric models, and break supersymmetry dynamically. This would require constructing orientiworld models with certain additional sectors that would dynamically break supersymmetry and then mediate it to the Standard Model sector. Alternatively, especially in scenarios with M_s of order TeV, we could consider constructing orientiworld backgrounds that are non-supersymmetric already at the tree level. Such models were originally discussed in [50] (for subsequent developments see [51]). Thus, one can construct anomaly-free chiral models where all tadpoles are canceled. In particular, these models contain anti-D3-branes (or anti-O3-planes). However, in all of such models we have *twisted* closed string tachyons. Let us point out that in the models of [50] all divergences due to the closed string tachyons coupling to the D-branes and orientifold planes are canceled just as all the massless tadpoles. So the presence of the closed string tachyons does not destabilize the gauge theories on D3-branes. As to the bulk, there is a possibility that, once all the tachyons condense, we obtain a supersymmetric closed string background [52]. If so, this would mean that we could construct orientiworld models where the bulk is supersymmetric, while the brane supersymmetry is broken at the tree level. In other words, the branes would be non-BPS, and would break all supersymmetries.

Such orientiworld models could also be interesting for the cosmological constant problem. In particular, if the bulk supersymmetry is intact, one might hope that it might protect the brane cosmological constant [53, 54, 55, 56]. Thus, if we consider a codimension-2 or higher brane in infinite-volume extra space, we have flat brane solutions for a continuous range of the brane tension (that is, unlike in the codimension-1 case, positive brane tension does not necessarily imply inflation on the brane). This by itself, however, is not sufficient to successfully address the cosmological constant problem. Indeed, even in higher codimension cases we do have solutions with inflation on the brane. The idea then is the following [53, 54]. Suppose the bulk is supersymmetric, and we break brane supersymmetry. Since the volume of the extra space is infinite, the bulk supersymmetry is, nonetheless, intact. Now, the cosmological constant is an infra-red quantity — it is measured at large distances. In the scenarios we discussed above, gravity becomes higher dimensional around the Hubble size, so, at least naively, we would then be measuring the higher-dimensional cosmological constant (and not the four-dimensional one), which vanishes[36] due to the unbroken bulk supersymmetry. It would be interesting to test this idea in higher codimension models.[37]

Another point we would like to comment on here is gauge coupling unification. Thus, in the scenario where M_s is of order TeV we might expect that the extrapolated unification of gauge couplings in the MSSM would have to be an accident. Here we would like to point out that this might not necessarily be the case. First, one might consider a scenario where the Standard Model fields live on D4- or D5-branes with the corresponding compactification sizes along the branes of order of $1/M_s$. Then we could have accelerated unification as in [57, 58, 59]. However, *a priori* there is another possibility here, which need *not* involve compactification. The general idea was originally discussed in [60] (also see [61]), and here we would like to tailor it for the orientiworld context. As we have already mentioned, in the scenario discussed in subsection 4.4 we have two different cut-offs on D3-branes, namely, $\Lambda \sim M_s$ and $\Lambda' \sim \widehat{M}_P$. The key reason why this is possible is that we have *two* extra dimensions transverse to the D3-branes that are untouched by $\mathcal{N} = 2$ supersymmetric orbifold twists. If these dimensions were compact, then we could have large logarithmic threshold corrections due to the corresponding open string windings [24]. The compactification in this case plays the role of an ultra-violet cut-off. In the orientiworld context, once we deform to a conifold, we effectively have such a cut-off, namely, Λ', even without compactification. Then one might hope that we could achieve unifica-

[36]Note that we could still obtain the accelerated Universe as in [36].

[37]This idea was tested in [56] in codimension-1 cases. It appears that in these cases either the brane world-volume theory is conformal, or gravity actually becomes four-dimensional again at the Hubble size in the presence of non-vanishing cosmological constant on the brane.

tion along the lines of [60] by having the dilaton and/or the relevant twisted field vacuum expectation values varying from point to point in the bulk (in some cases such backgrounds could be studied in the F-theory [62] language [60]). It would be interesting to see if this idea can be made more concrete.

Finally, let us mention that various issues such as proton stability, flavor violation as well as neutrino masses might need to be addressed in the orientiworld context with M_s of order TeV. It is important to note that gravity in such scenarios is weak. Nonetheless, one might have to worry about the open string excitations in this context. Some of the ideas discussed in the context of [2] might be applicable in the orientiworld context as well. In particular, proton stabilization [63, 64], neutrino masses [65, 64] and flavor violation [66, 67, 68] can be addressed in theories with M_s of order TeV.

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A. The U(1) factors in orientiworld models

In this appendix we give the calculation of the tree-level exchange of a twisted 2-form in the cylinder amplitude that leads to renormalization of the U(1) gauge couplings in orientiworld models. More precisely, as we already mentioned in subsection 2.3, for our purposes here it suffices to discuss this calculation in the oriented type-IIB setup. In fact, for simplicity we will focus on the $\mathbb{Z}_2$ model discussed in subsection 2.3.

The relevant action reads (see subsection 2.3 for details):

$$-\frac{L^2}{12}\int_{\rm D3}\overline{H}_R^2+\int_{\rm D3}\overline{C}_R\wedge F_- - \frac{1}{4g_-^2}\int_{\rm D3}F_-^2-\frac{1}{12}\int_{{\rm D3}\times\mathbb{R}^2}H_R^2\,. \tag{A.1}$$

What we need to compute is the exchange of the 2-form C_R between two boundaries with a $\mathrm{U}(1)_-$ gauge boson attached to each boundary. To do this, we can simply compute the C_R field created by a $\mathrm{U}(1)_-$ background field, and then couple it back to this background field. That is, we are simply computing the classical self-interaction of the $\mathrm{U}(1)_-$ field strength via the C_R field.

We need to fix the gauge for the C_R field. A convenient gauge choice is

$$\partial_M C_R^{MN}=0\,. \tag{A.2}$$

The gauge-fixed equation of motion for C_R is then given by (here we are taking into account the fact that $C_R^{\mu i} = C_R^{ij} = 0$):

$$-\partial^K \partial_K C_R^{MN} = \delta^M{}_\mu \delta^N{}_\nu \left[2\epsilon^{\mu\nu}{}_{\sigma\rho} F_-^{\sigma\rho} + L^2 \partial^\sigma \partial_\sigma C_R^{\mu\nu}\right] \delta^{(2)}(x^i)\,. \tag{A.3}$$

It is convenient to Fourier transform the coordinates x^μ and x^i. Let the corresponding momenta be p^μ and k^i, respectively. We then have the following equation for $C_R^{MN}(p, x^i)$:

$$C_R^{MN}(p, x^i) = \delta^M{}_\mu \delta^N{}_\nu \left[2\epsilon^{\mu\nu}{}_{\sigma\rho} F_-^{\sigma\rho}(p) - L^2 p^2 C_R^{\mu\nu}(p, 0)\right] \int \frac{d^2k}{(2\pi)^2} \frac{e^{-ik\cdot x}}{k^2 + p^2}\,. \tag{A.4}$$

In particular, we have:

$$\overline{C}_R^{\mu\nu}(p) = \left[2\epsilon^{\mu\nu}{}_{\sigma\rho} F_-^{\sigma\rho}(p) - L^2 p^2 \overline{C}_R^{\mu\nu}(p)\right] \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + p^2}\,. \tag{A.5}$$

We must regularize the integral over k^i in the ultra-violet. We will use the following regularization:

$$\int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + p^2} = \int^{k^2 = \widetilde{\Lambda}^2} \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + p^2} = \frac{1}{4\pi} \ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)\,. \tag{A.6}$$

Note that $\widetilde{\Lambda}$ here is the ultra-violet cut-off in the *tree channel*, which corresponds to an infra-red cut-off in the *loop channel*. We then have:

$$\overline{C}_R^{\mu\nu}(p) = \frac{1}{2\pi} \epsilon^{\mu\nu}{}_{\sigma\rho} F_-^{\sigma\rho}(p) \, \frac{\ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}{1 + \frac{L^2p^2}{4\pi} \ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}\,. \tag{A.7}$$

The self-interaction term is given by:

$$\int_{\mathrm{D3}} \overline{C}_R \wedge F_- = -\frac{2}{\pi} \int \frac{d^4p}{(2\pi)^4} \, F_-^{\mu\nu}(p) F_{-\mu\nu}(-p) \, \frac{\ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}{1 + \frac{L^2p^2}{4\pi} \ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}\,. \tag{A.8}$$

This results in the renormalization of the $\mathrm{U}(1)_-$ gauge coupling. The renormalized kinetic term for the $\mathrm{U}(1)_-$ gauge field is now given by:

$$-\int \frac{d^4p}{(2\pi)^4} \frac{1}{4\widetilde{g}_-^2(p)} \, F_-^{\mu\nu}(p) F_{-\mu\nu}(-p)\,, \tag{A.9}$$

where

$$\frac{1}{\widetilde{g}_-^2(p)} = \frac{1}{g_-^2} + \frac{8}{\pi} \, \frac{\ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}{1 + \frac{L^2p^2}{4\pi} \ln\left(1 + \frac{\widetilde{\Lambda}^2}{p^2}\right)}\,. \tag{A.10}$$

For $L = 0$ this reproduces the usual logarithmic $\mathrm{U}(1)_-$ gauge coupling running (in fact, it is not difficult to check that with the normalization for the bare $\mathrm{U}(1)_-$ gauge coupling g_- we have adopted, we have the correct one-loop β-function coefficient). In this case we have a logarithmic divergence, which must be regularized by introducing the tree-channel ultra-violet cut-off $\widetilde{\Lambda}$. We also have a logarithmic tree-channel infra-red divergence at $p^2 \to 0$, which translates into the corresponding loop-channel ultra-violet divergence.

However, for non-vanishing L the gauge coupling does *not* run. In fact, we only have a *finite non-local* correction even if we take $\widetilde{\Lambda} \to \infty$ limit (that is, if we remove the tree-channel ultra-violet cut-off):

$$\frac{1}{\widetilde{g}_-^2(p)} = \frac{1}{g_-^2}\left[1 + \frac{2m_-^2}{p^2}\right] . \tag{A.11}$$

Here it is important to note that we do *not* have a tree-channel infra-red divergence either. In particular, the integral over p^μ in (A.9) is cut off in the infra-red at $p^2 = m_-^2$, where

$$m_-^2 = \frac{16 g_-^2}{L^2} \tag{A.12}$$

is the mass of the $\mathrm{U}(1)_-$ gauge boson. That is, we do not have a loop-channel ultra-violet divergence either.

Thus, as we see, once we introduce the kinetic term for the C_R field on the D3-branes, we have no ultra-violet or infra-red divergences in the $\mathrm{U}(1)_-$ coupling. As we argued in subsection 2.3, such a kinetic term would be generated at the one-loop level with

$$L^2 = b(\alpha')^2\Lambda^2 , \tag{A.13}$$

where b is a dimensionless numerical coefficient, and Λ is the loop-channel ultra-violet cut-off. We then interpret the above result as the $\mathrm{U}(1)_-$ symmetry being *global* and not local.

In fact, here we would like to argue that this is consistent with the case where the two extra dimensions x^i are compactified on a large 2-torus. Thus, consider a model where we compactify x^i on a square 2-torus T^2:

$$x^i \sim x^i + 2\pi r . \tag{A.14}$$

Then in the absence of Wilson lines on T^2 we actually have the same four-dimensional spectrum. Let us assume that $r^2 \gg \alpha'$. Then we actually have a *tree-level* 4-dimensional kinetic term (2.21) for the C_R field with

$$L^2 = (2\pi r)^2 . \tag{A.15}$$

Thus, the $\mathrm{U}(1)_-$ gauge field is massive. In fact, we can now see why (A.13) should be the case. Note that in the D3-brane language we have *winding* open string modes with masses:

$$M_w^2 = w^2 \frac{r^2}{(\alpha')^2}\,, \tag{A.16}$$

where $w^2 = (w_1)^2 + (w_2)^2$, and $w_i \in \mathbb{Z}$. This can also be seen by T-dualizing along the T^2. The D3-branes then go into D5-branes, and the 2-torus T^2 goes into a dual 2-torus $\widetilde{T}^2$ with the coordinates

$$\widetilde{x}^i \sim \widetilde{x}^i + 2\pi\widetilde{r}\,, \tag{A.17}$$

where

$$\widetilde{r} = \frac{\alpha'}{r}\,. \tag{A.18}$$

Note that $\widetilde{r}^2 \ll \alpha'$. Now we have Kaluza-Klein modes with masses

$$M_m^2 = \frac{m^2}{\widetilde{r}^2}\,, \tag{A.19}$$

where $m^2 = (m_1)^2 + (m_2)^2$, and $m_i \in \mathbb{Z}$. These Kaluza-Klein modes in the D5-brane language correspond to the winding modes in the D3-brane language.

Note that the theory on the D5-branes is a 4-dimensional gauge theory at the momenta $p^2 \ll 1/\widetilde{r}^2$ (that is, below the Kaluza-Klein threshold). Even though the string excitations lie way below the Kaluza-Klein threshold, they do not contribute to the gauge coupling renormalization as we discussed in subsection 2.3. This implies that the 4-dimensional gauge theory cut-off is given by the Kaluza-Klein threshold [17, 24, 25] (that is, compactification on T^2 effectively introduces an ultra-violet cut-off into the D3-brane gauge theory):

$$\Lambda = \frac{\xi}{\widetilde{r}} = \frac{\xi r}{\alpha'}\,, \tag{A.20}$$

where a dimensionless numerical coefficient ξ parametrizes the subtraction scheme dependence. This confirms our intuition that we should have (A.13). Note that at the momenta $p^2 \gg 1/\widetilde{r}^2$ the D5-brane theory becomes 6-dimensional. In this 6-dimensional theory (which can be thought about in the language of a model where we place non-compact D5-branes at the $\mathbb{Z}_2$ orbifold singularity) the $\mathrm{U}(1)_-$ gauge symmetry is anomalous (note that in six dimensions U(1) anomalies come from box diagrams). It is broken by a Chern-Simons mixing with the twisted 4-form, call it C'_R:

$$\int_{\mathrm{D5}} C'_R \wedge F_-\,. \tag{A.21}$$

Note that this twisted 4-form propagates in six dimensions along the D5-brane world-volume. Its exchange in the cylinder amplitude with a $\mathrm{U}(1)_-$ gauge boson attached to each boundary gives rise to a quadratic divergence. However, this quadratic divergence is removed once $\mathrm{U}(1)_-$ acquires mass, which breaks the $\mathrm{U}(1)_-$ gauge symmetry to a global one.

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B. A completely smooth brane

In this appendix we would like to discuss the case where we smooth out the brane completely, that is, when we replace the d-dimensional δ-function $\delta^{(d)}(x^i - x^i_*)$ by a smooth distribution $f^{(d)}(x^i - x^i_*)$. In fact, for our purposes here it will suffice to study the simple distribution $f_1(r)$ given by (3.29).

Thus, consider the equation of motion for χ in the absence of the brane matter (see subsection 3.3 for details):

$$\left(\partial^i\partial_i - p^2\right)\chi = -\kappa L^d p^2 \chi f_1(r)\,. \tag{B.1}$$

We have a continuum of modes with $p^2 \leq 0$ corresponding to massless and massive modes (here we are working with Minkowski momenta p^μ). However, we also have an infinite tower of quadratically normalizable tachyonic modes. Thus, let us assume that $p^2 > 0$. Let

$$q^2 \equiv p^2 \left[\frac{\kappa L^d}{v_d \epsilon^d} - 1\right] \equiv \gamma^2 p^2 > 0\,. \tag{B.2}$$

Then the equation of motion for χ reads:

$$\begin{aligned} &\left(\partial^i\partial_i + q^2\right)\chi = 0\,, \qquad r \leq \epsilon\,,\\ &\left(\partial^i\partial_i - p^2\right)\chi = 0\,, \qquad r > \epsilon\,. \end{aligned} \tag{B.3}$$

As before, for simplicity let us focus on the $d = 3$ case. Then the radially symmetric solution is given by:

$$\begin{aligned} \chi(r) &= C\,\frac{\sin(qr)}{r}\,, \qquad r \leq \epsilon\,,\\ \chi(r) &= C\,\sin(p\epsilon)\,\frac{\exp\left(-p[r-\epsilon]\right)}{r}\,, \qquad r > \epsilon\,, \end{aligned} \tag{B.4}$$

where C is a constant, and the matching of $\partial_r\chi(r)$ at $r = \epsilon$ gives the following condition on p:

$$\tan(q\epsilon) = -\frac{q}{p}\,. \tag{B.5}$$

This equation has infinitely many roots (here we must keep those corresponding to $p > 0$):

$$p_n = \frac{\pi}{2\gamma\epsilon}\,(2n-1) + \frac{1}{\gamma\epsilon}\arctan\left(\frac{1}{\gamma}\right)\,, \qquad n \in \mathbb{N}\,. \tag{B.6}$$

If the brane thickness is small ($\epsilon \ll L$), then $\gamma \gg 1$, and we have

$$p_n \approx \frac{\pi}{2\gamma\epsilon}\,(2n-1)\,, \qquad n \in \mathbb{N}\,. \tag{B.7}$$

Thus, we have an infinite tower of quadratically normalizable tachyonic modes in this case.

C. Graviton propagator on the brane

In the main text we studied the $d = 3$ case in detail. However, we can equally easily determine the graviton propagator on the brane in general d. Because of the logarithmic property of the 2-dimensional propagator, the $d = 2$ case requires a separate treatment, while the $d > 2$ cases can be discussed together. In this appendix we provide the corresponding details.

C.1 The $d > 2$ cases

For the reader's convenience, here we repeat the relevant equations:

$$\begin{aligned}\overline{H}_{\mu\nu} \equiv H_{\mu\nu} - \frac{1}{D-d}\eta_{\mu\nu}H = -M_P^{2-D}\Bigg[\left(T_{\mu\nu}(p) - \frac{1}{D-d}\eta_{\mu\nu}T(p)\right)\Phi + \\ + \frac{d}{D-2}L^d\left(p_\mu p_\nu - \frac{1}{D-d}\eta_{\mu\nu}p^2\right)T(p)\Sigma\Bigg]\,, \end{aligned} \tag{C.1}$$

$$H = -\frac{d-2}{d}\chi = -\frac{d-2}{D-2}M_P^{2-D}T(p)\widetilde{\Phi}\,. \tag{C.2}$$

Here Φ, Σ and $\widetilde{\Phi}$ are the solutions to the following equations

$$\begin{aligned}\left(\partial^i\partial_i - p^2\right)\Phi &= \left[1 + L^d p^2\Phi\right] f(r)\,, \\ \left(\partial^i\partial_i - p^2\right)\Sigma &= \left[\widetilde{\Phi} + L^d p^2\Sigma\right] f(r)\,, \\ \left(\partial^i\partial_i - p^2\right)\widetilde{\Phi} &= \left[1 - \kappa L^d p^2\widetilde{\Phi}\right] f(r)\end{aligned} \tag{C.3}$$

subject to the condition that Φ, Σ and $\widetilde{\Phi}$ decay to zero away from the brane (here we are focusing on the cases $d > 2$, where this is true even for the $p^2 = 0$ modes). As before, we are using the notation

$$\kappa \equiv \frac{(d-1)(D-d-2)}{D-2} \tag{C.4}$$

to simplify expressions containing $\widetilde{\Phi}$.

Let us solve for $H_{\mu\nu}$ and χ on the brane, that is, at $r = \epsilon$. Here we will assume that $\epsilon \ll L$. Consider the momenta with $p^2 \ll 1/\epsilon^2$. Then it is not difficult to see that, up to $\mathcal{O}(p\epsilon)$ corrections, we have:

$$\begin{aligned}\Phi(r) &\approx A\,, \qquad r \leq \epsilon\,, \\ \Phi(r) &\approx \frac{B}{r^{d-2}}\,, \qquad r \gtrsim \epsilon\,,\end{aligned} \tag{C.5}$$

where A and B are constants. Matching the values of $\Phi(r)$ at $r = \epsilon$ gives

$$B \approx \epsilon^{d-2}A\,. \tag{C.6}$$

On the other hand, from (C.3) we obtain:

$$-(d-2)\frac{B}{\epsilon^{d-1}} \approx \left[1+L^d p^2 A\right]\frac{1}{a_{d-1}\epsilon^{d-1}}\,. \tag{C.7}$$

We, therefore, have

$$\Phi(\epsilon) \approx -\frac{1}{L^d}\,\frac{1}{p^2+p_c^2}\,, \tag{C.8}$$

where

$$p_c^2 \equiv \frac{(d-2)a_{d-1}\epsilon^{d-2}}{L^d}\,. \tag{C.9}$$

Similarly, for $\widetilde{\Phi}$ and Σ we have:

$$\begin{aligned}\widetilde{\Phi}(\epsilon) &\approx \frac{1}{\kappa L^d}\,\frac{1}{p^2-p_*^2}\,,\\ \Sigma(\epsilon) &\approx -\frac{1}{\kappa L^{2d}}\,\frac{1}{p^2+p_c^2}\,\frac{1}{p^2-p_*^2}\,,\end{aligned} \tag{C.10}$$

where

$$p_*^2 \equiv \frac{(d-2)a_{d-1}\epsilon^{d-2}}{\kappa L^d}\,. \tag{C.11}$$

On the brane we then have:

$$\begin{aligned}H_{\mu\nu}(r=\epsilon) \approx \frac{\widehat{M}_P^{2+d-D}}{p^2+p_c^2}\Bigg[T_{\mu\nu}(p) - \frac{1}{D-d-2}\eta_{\mu\nu}T(p) + &\qquad (\text{C.12})\\ +\frac{d}{(d-1)(D-d-2)}\left(p_\mu p_\nu - \eta_{\mu\nu}\frac{(d-2)p_c^2+2(d-1)p_*^2}{d(D-d)}\right)\frac{T(p)}{p^2-p_*^2}\Bigg]\,,&\\ \chi(r=\epsilon) \approx \frac{d}{(d-1)(D-d-2)}\,\frac{\widehat{M}_P^{2+d-D}}{p^2-p_*^2}\,T(p)\,. &\qquad (\text{C.13})\end{aligned}$$

Note that at the momenta $p^2 \gg p_c^2$ the graviton propagator on the brane is $(D-d)$-dimensional.

C.2 The $d=2$ case

Once again, let us assume that $\epsilon \ll L$, and consider the momenta with $p^2 \ll 1/\epsilon^2$. Then it is not difficult to see that, up to $\mathcal{O}(p\epsilon)$ corrections, we have:

$$\begin{aligned}\Phi(r) &\approx A\,, \qquad r \leq \epsilon\,,\\ \Phi(r) &\approx B\ln\left(\frac{r}{R}\right), \qquad r \gtrsim \epsilon\,,\end{aligned} \tag{C.14}$$

where A, B and R are constants. Matching the values of $\Phi(r)$ at $r=\epsilon$ gives

$$B \approx A\ln^{-1}\left(\frac{\epsilon}{R}\right). \tag{C.15}$$

On the other hand, from (C.3) we obtain:

$$\frac{B}{\epsilon} \approx \left[1 + L^2 p^2 A\right] \frac{1}{2\pi\epsilon} \,. \tag{C.16}$$

We, therefore, have

$$\Phi(\epsilon) \approx -\frac{1}{L^2} \, \frac{1}{p^2 + p_c^2} \,, \tag{C.17}$$

where

$$p_c^2 \equiv \frac{2\pi}{L^2} \, \ln^{-1}\left(\frac{R}{\epsilon}\right) . \tag{C.18}$$

Similarly, for $\widetilde{\Phi}$ we have:

$$\begin{aligned} \widetilde{\Phi}(\epsilon) &\approx \frac{1}{\kappa L^2} \frac{1}{p^2 - p_*^2} \,, \\ \Sigma(\epsilon) &\approx -\frac{1}{\kappa L^4} \, \frac{1}{p^2 + p_c^2} \, \frac{1}{p^2 - p_*^2} \,, \end{aligned} \tag{C.19}$$

where

$$p_*^2 \equiv \frac{2\pi}{\kappa L^2} \, \ln^{-1}\left(\frac{\xi R}{\epsilon}\right) , \tag{C.20}$$

where ξ is a dimensionless parameter. On the brane we then have:

$$\begin{aligned} H_{\mu\nu}(r=\epsilon) \approx \frac{\widehat{M}_P^{4-D}}{p^2 + p_c^2} \Bigg[& T_{\mu\nu}(p) - \frac{1}{D-4} \eta_{\mu\nu} T(p) + \\ & + \frac{2}{D-4} \left(p_\mu p_\nu - \eta_{\mu\nu} \frac{p_*^2}{D-2} \right) \frac{T(p)}{p^2 - p_*^2} \Bigg] \,, \end{aligned} \tag{C.21}$$

$$\chi(r=\epsilon) \approx \frac{2}{D-4} \, \frac{\widehat{M}_P^{4-D}}{p^2 - p_*^2} \, T(p) \,. \tag{C.22}$$

Note that $H_{\mu\nu}$ is traceless in this case. Also, note that at the momenta $p^2 \gg p_c^2$ the graviton propagator on the brane is $(D-2)$-dimensional. If we take $R \sim L$, then we can have $p_c \ll 1/L$. Note, however, that in this case p_c is only logarithmically small compared with $1/L$. On the other hand, if we take $R \to \infty$, then $p_c, p_* \to 0$, and there is no cross-over between the $(D-2)$-dimensional gravity and the D-dimensional gravity (that is, gravity on the brane is always $(D-2)$-dimensional). Note that R is a tree-level infra-red cut-off. We can then take the $R \to \infty$ limit in the cases where such a cut-off is absent.

D. A non-zero tension codimension-2 brane

In this appendix we discuss the case of a codimension-2 brane with positive tension. In $d = 2$ the equations of motion for the warp factors A and B discussed in subsection 3.4 simplify as follows:

$$
\begin{aligned}
2A'B' - (A')^2 - A'' + \frac{1}{r}A' &= 0\,,\\
A'' + \frac{1}{2}(D-1)(A')^2 - A'B' &= 0\,,\\
(D-3)\left[A'' + \frac{1}{2}(D-2)(A')^2 + \frac{1}{r}A'\right] + B'' + \frac{1}{r}B' + \frac{1}{2}\widehat{\tau} f(r) &= 0\,.
\end{aligned}
\tag{D.1}
$$

It is then not difficult to see that for non-zero $\widehat{\tau}$ the only solution is given by $A \equiv 0$ (more precisely, A should be constant, which we can set to zero), and

$$
B'' + \frac{1}{r}B' + \frac{1}{2}\widehat{\tau} f(r) = 0\,. \tag{D.2}
$$

The smooth solution to this equation is given by (we have set the integration constant so that B vanishes on the brane):

$$
B(r) = -\lambda\theta(r-\epsilon)\ln\left(\frac{r}{\epsilon}\right)\,, \tag{D.3}
$$

where

$$
\lambda \equiv \frac{\widehat{\tau}}{4\pi}\,. \tag{D.4}
$$

Note that the space is locally flat, but for $r > \epsilon$ we have a deficit angle equal

$$
2\pi\lambda = \frac{1}{2}\widehat{\tau}\,. \tag{D.5}
$$

We will therefore assume that $0 < \widehat{\tau} < 4\pi$.

In this background we can also solve the linearized equations of motion for the graviton and the graviscalars [32]. Thus, we have (note that $H_{\mu\nu}$ is traceless):

$$
\begin{aligned}
H_{\mu\nu} = &-M_P^{2-D}\left[\left(T_{\mu\nu}(p) - \frac{1}{D-2}\eta_{\mu\nu}T(p)\right)\Phi + \right.\\
&\left. + \frac{2}{D-2}L^2\left(p_\mu p_\nu - \frac{1}{D-2}\eta_{\mu\nu}p^2\right)T(p)\Sigma\right]\,,
\end{aligned}
\tag{D.6}
$$

$$
\chi = \frac{2}{D-2}M_P^{2-D}T(p)\widetilde{\Phi}\,. \tag{D.7}
$$

Here Φ, Σ and $\widetilde{\Phi}$ are the solutions to the following equations ($\kappa = (D-4)/(D-2)$):

$$
\begin{aligned}
\left(\partial^i\partial_i - e^{2B}p^2\right)\Phi &= \left[1 + L^2p^2\Phi\right] f(r)\,,\\
\left(\partial^i\partial_i - e^{2B}p^2\right)\Sigma &= \left[\widetilde{\Phi} + L^2p^2\Sigma\right] f(r)\,,\\
\left(\partial^i\partial_i - e^{2B}p^2\right)\widetilde{\Phi} &= \left[1 - \kappa L^2p^2\widetilde{\Phi}\right] f(r)\,.
\end{aligned}
\tag{D.8}
$$

It is not difficult to see that the e^{2B} factor does not affect the discussion of appendix C.2 for momenta $p^2 \ll 1/\epsilon^2$ (assuming that $\epsilon \ll L$). In particular, all the equations and conclusions given there are unmodified up to $\mathcal{O}(p\epsilon)$ corrections.

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